First Name_____ Last Name_____ NetID_____

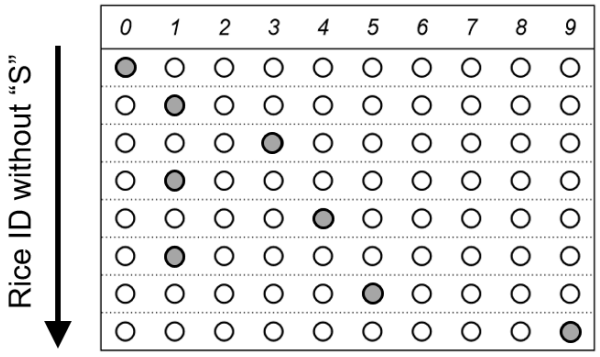
Physics 102 Spring 2023 Final

Time allowed: 3 hours, closed book

Instructions:
Please print your name and NetID in **two** places: on the top of this cover sheet, and on the multiple-choice answer sheet. If you use any additional paper for your work, please make sure to print your name on the top of each sheet and staple the extra sheets to this exam packet when you hand in your work at the end of the session. **(No name, No credit!)**

- There are 15 multiple choice questions and 4 free response questions in total.
- The maximum possible points are 140 points.
- Mark your answers to the multiple-choice questions on the answer sheet provided. Make sure to fill the appropriate bubble **completely** using a #2 pencil, or a black pen. Any multiple-choice responses written on pages other than the answer sheet will NOT be graded.
- Write all your solutions to the free response questions in the space provided in the exam packet, or on the extra space provided at the end of the exam packet. Make sure that it is very clear which problem your work corresponds to. If needed, extra paper will be provided at the front of the exam room. Remember to print your name on any extra sheets and staple them to the exam packet.
- When you finish, please place the exam packets and the multiple-choice answer sheet in two separate piles at the front of the exam room. If you used additional sheets of paper, make sure to staple them to the exam packet. Hand in all your work at the end of the 90-minute exam period.
- You are not allowed to take anything written away from the exam room.
- You may not use phones, computers, tablets, or any other web connected device during the exam.
- You may not use the symbolic manipulation or graphing capabilities of your calculator. (You can look up trig functions, *i.e.*, calculating sin(45) is not a symbolic manipulation.)
- You may not store, or use pre-stored formulae, saved in your calculator's memory, or anything written down in advance of entering the exam.

On your multiple-choice answer sheet, you will need to fill in your Rice ID. Beginning with the numbers “01”, enter your Rice ID by bubbling in one number per row. In the example below, the Rice ID entered is "S01314159".



Potentially Useful Constants and Integrals:

$$g = 9.8\ \frac{\text{m}}{\text{s}^2} \qquad e = 1.602 \times 10^{-19}\ \text{C}$$

$$\varepsilon_0 = 8.854 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2} \qquad m_e = 9.109 \times 10^{-31}\ \text{kg}$$

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \qquad m_P = 1.673 \times 10^{-27}\ \text{kg}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (\text{for } n \neq -1) \qquad \int \frac{dx}{(a-x)^2} = \frac{1}{a-x}$$

$$\int \frac{dx}{x} = \ln x \qquad \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax \qquad \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}}$$

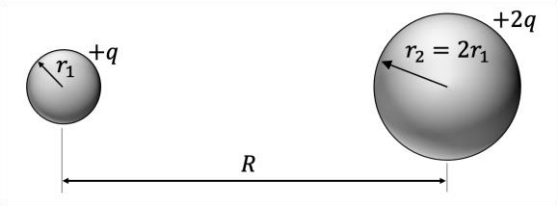
$$\int e^{ax}dx = \frac{1}{a}e^{ax} \qquad \int \frac{x\,dx}{(x^2 \pm a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$$

$$\int \frac{dx}{a-x} = -\ln(a-x)$$

Multiple Choice Questions (4 points each):

1. Consider two spheres of radii r_1 and $r_2 = 2r_1$, whose centers are separated by $R \gg r_1, r_2$ and that carry charges of $Q_1 = q$ and $Q_2 = 2q$ respectively. The magnitude of the force between them is F. If a conductive wire is connected between the spheres and then removed, the magnitude of the force between them becomes _____.

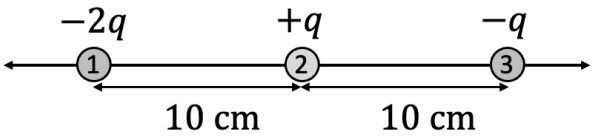
   a) $F/3$
   b) $F/2$
   c) $2F/3$
   d) F
   e) $3F/2$



   (Hint: You can assume the wire is insulated and carries no net charge upon removal, *i.e.*, no change in the total charge Q_1 plus Q_2 occurs.)

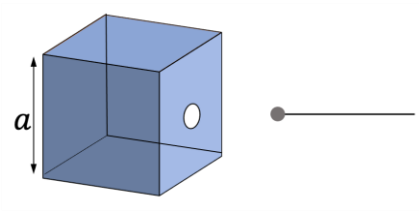
2. Consider three charges, $Q_1 = -2q, Q_2 = q$, and $Q_3 = -q$, positioned as shown. Which of the following best describes the directions of the forces experienced by Q_1, Q_2, and Q_3 (in that order)?

   a) zero, right, left.
   b) right, left, right.
   c) right, left, left.
   d) right, right, left.
   e) right, right, zero.



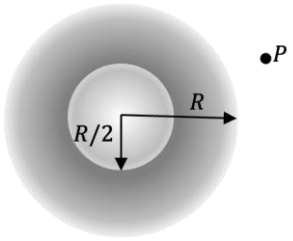
3. Consider an isolated cubical hollow metal box of side a that initially carries a net charge of $+3q$. If a charge $-3q$ is introduced at the center of the box without touching the box, the net charge on the outer surface of the box is now _____. (*You should assume that the size of the hole in the side of the box is negligibly small.*)

   a) $9q$.
   b) $6q$.
   c) $3q$.
   d) 0.
   e) This cannot be determined without knowing a.

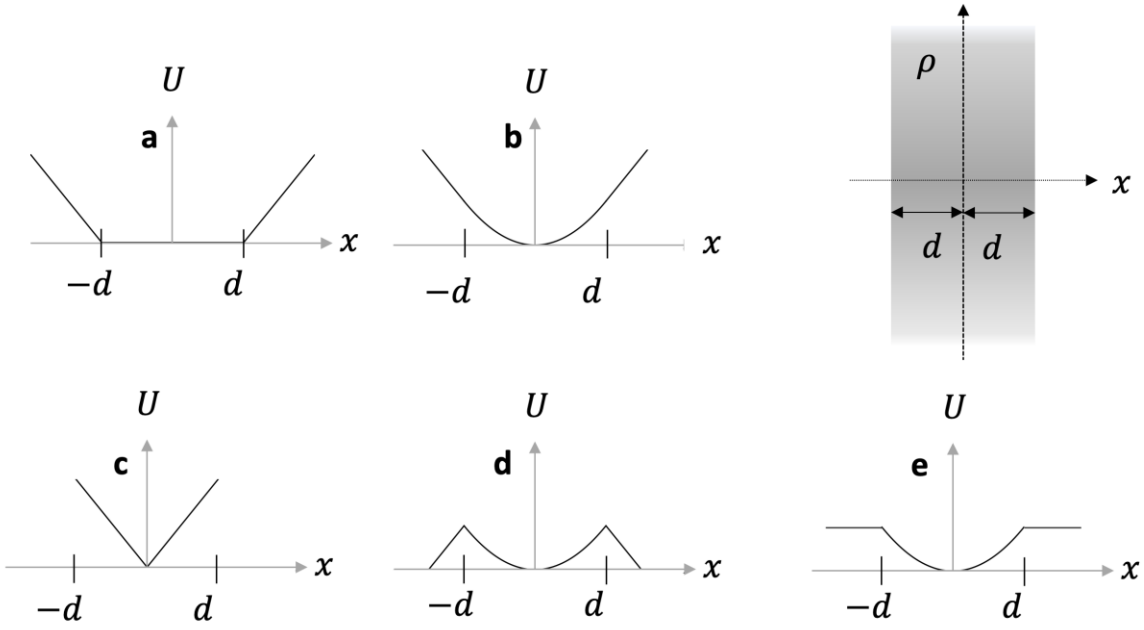


4. Consider a spherical cloud of charge of radius R with a uniform charge density ρ which produces an electric field E at a point P outside the sphere. If the central portion of the charge with radius $R/2$ is removed to leave a void, the electric field at P due to the remaining charge becomes _____.

   a) $7E/8$
   b) $3E/4$
   c) $E/2$
   d) $E/4$
   e) $E/8$

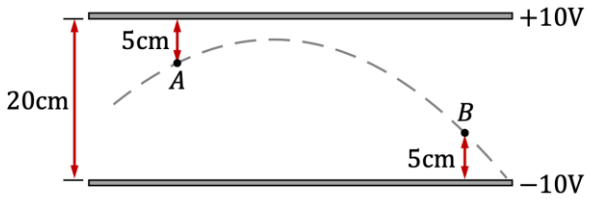


5. Consider a very large planar slab of charge of thickness $2d$ and uniform charge density ρ that lies in the yz-plane. Which of the following graphs best describes the variation in potential along the x-axis? (Assume that the potential at $x = 0$ is 0.)

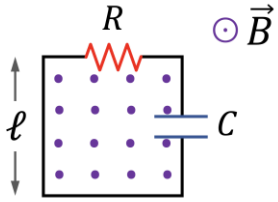


6. Consider a large parallel-plate capacitor with plates separated by $d = 20$ cm that are held at potentials of $\pm 10\ V$. A particle with charge $+7C$ is fired into the gap and follows the trajectory shown. In moving from point A, which is 5 cm from the upper plate, to point B, which is 5 cm from the lower plate, the change in kinetic energy of the particles is _____.

a) 210 J
b) 140 J
c) 70 J
d) 35 J
e) 20 J

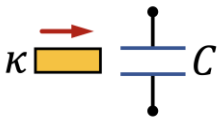


7. Consider the arrangement shown comprising a resistor with $R = 100\ \text{k}\Omega$ and a capacitor C. The two are connected in series to form a square of side $\ell = 20$ cm. At $t = 0$ the circuit begins to experience a uniform magnetic field directed out of the plane of the circuit that increases steadily at a rate of $0.0005\ \text{T/s}$ and continues to do so for a very long time. If it is observed that the voltage across the capacitor reaches $1/e$ of its final maximum value in 15 s, the value of C is approximately _____.



a) 150 μF
b) 330 μF
c) 620 μF
d) 740 μF
e) None of the above - the charge on the capacitor remains zero.

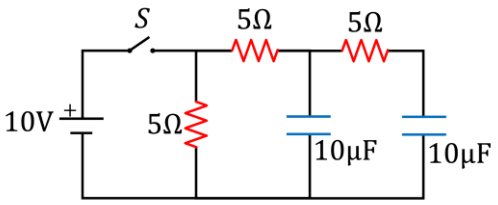
8. Consider an air-spaced parallel-plate capacitor that has been charged up by connecting it to a battery which is then removed, leaving the capacitor isolated. If an insulator with dielectric constant $\kappa = 2$ is slid in between the plates, completely filling the space between the plates, this will _____.



a) halve the capacitance.
b) double the energy stored in the capacitor.
c) double the surface charge density on each plate.
d) halve the electric field between the plates.
e) double the attractive force on each plate.

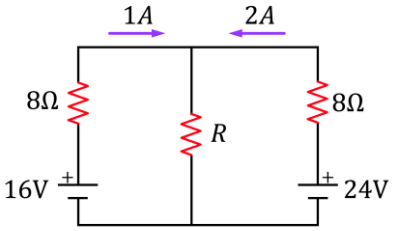
9. Consider the circuit shown below. Switch S has been open for a long time and is then closed. The current through the source of emf is _____ immediately after switch S is closed.

a) 1 A
b) 3 A
c) 4 A
d) 5 A
e) 6 A



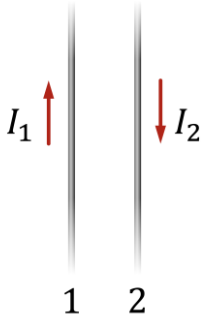
10. Consider the circuit shown which carries the currents indicated. What is the resistance R?

a) 2.67 Ω
b) 3.00 Ω
c) 4.33 Ω
d) 8.00 Ω
e) 9.33 Ω



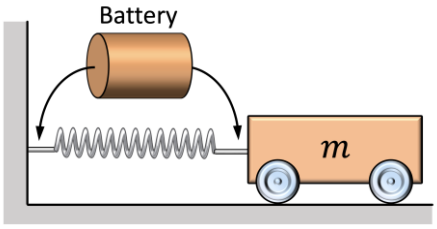
11. Consider two long straight parallel wires that carry currents of $I_1 = 4$ A, and $I_2 = 8$ A as shown. If the magnetic force exerted on wire 1 is F_1 and that on wire 2 is F_2, the relationship between their magnitudes Is:

a) $F_1 = 4F_2$
b) $F_1 = F_2/2$
c) $F_1 = F_2$
d) $F_1 = 2F_2$
e) $F_1 = F_2/4$



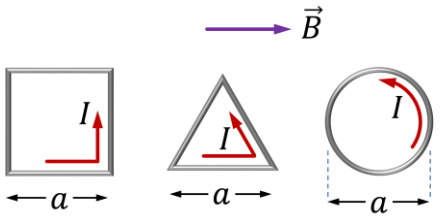
12. A friction-free cart of mass m is initially at rest and is attached to a long loosely-wound conducting spring at equilibrium. A large current is then directed through the spring by connecting its ends to a battery using flexible leads. Immediately following this, which of the following statement is true?

a) The cart remains at rest.
b) The cart is moving to the left.
c) The cart is moving to the right.
d) The direction of motion cannot be determined without knowing the direction of current flow.
e) The direction of motion cannot be determined without knowing whether the spring has a left- or right-hand spiral.



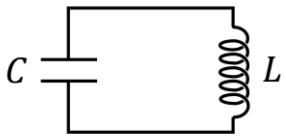
13. Three current-carrying loops are depicted below along with their dimensions. Each carries a current of the same magnitude as shown. If the loops are situated in a uniform magnetic field that is directed to the right, which loop experiences the greatest torque?

    a) The rectangular loop.
    b) The triangular loop.
    c) The circular loop.
    d) They all experience the same torque.
    e) They all experience zero torque.

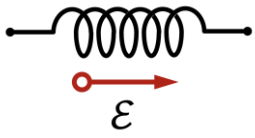


14. Consider the LC oscillator shown, where the inductor is in the form of along solenoid of length ℓ with N-many turns and radius $r \ll \ell$. If the solenoid is replaced by one of length 2ℓ but with the same radius r and number of turns N, the oscillation frequency of the circuit will _____.

    a) increase by a factor of 4
    b) increase by a factor of 2
    c) increase by a factor of $\sqrt{2}$.
    d) remain the same.
    e) This cannot be determined without knowing C and L.



15. The diagram shows an inductor that is part of a circuit. If at some instant of time the direction of the $\mathcal{E}$ induced in the inductor is as shown, we can conclude:

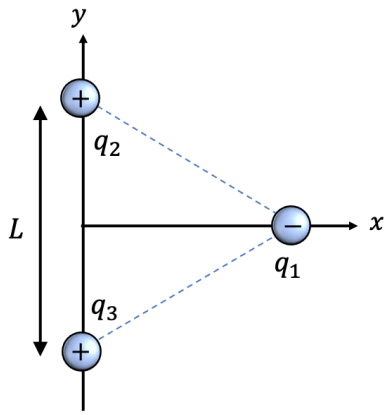


a) The current is flowing to the right and is constant.
b) The current is flowing to the left and is constant.
c) The current is flowing to the right and is increasing.
d) The current is flowing to the left and is decreasing.
e) None of the above conclusions are correct.

Free-Response Questions: (20 points each)

1. Three charges $q_1 = -2.00\ \mu\text{C}$ and $q_2 = q_3 = 3.00\ \mu\text{C}$ are held fixed at the corners of an equilateral triangle, as shown. The side length of the equilateral triangle is $L = 7.00\ \text{cm}$.

   Use the coordinate system given in the figure when indicating directions.



(a) What is the net electric force on the rightmost charge q_1?
Report your answer in unit-vector notation.

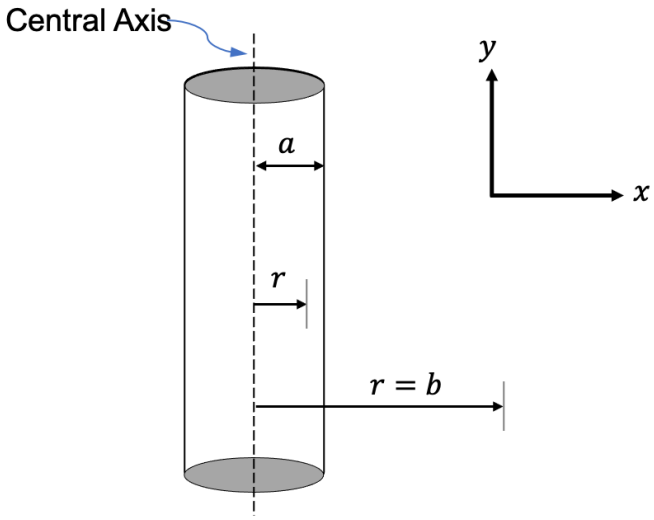
(b) What is the net electric field due to charges q_2 and q_3 at the location of charge q_1? *Report your answer in unit-vector notation.*

(1. continued)

(c) How much work was done against electric forces to assemble the entire system as shown?

(d) How much work must you do to move the charge q_1 from its original location to the origin of the xy-axes shown above?

2. Consider an infinitely-long positively-charged cylinder with a uniform positive volume charge density, $\rho > 0$, and radius a. The central axis of the cylinder is along the y-axis, as shown in the figure below.



Express your final answers to the questions in terms of the symbols ρ, a, b, Q, m, r, ε_0, π, and real numbers, as needed.

(a) What is the magnitude of the electric field due to the cylinder at an arbitrary distance $r < a$ from the central axis of the cylinder (*i.e.*, inside the cylinder)?

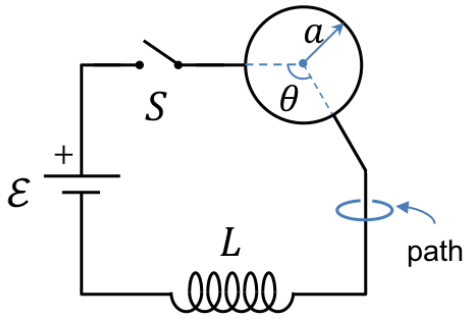
(b) What is the magnitude of the electric field due to the cylinder at an arbitrary distance $r > a$ from the central axis of the cylinder (*i.e.*, outside the cylinder)?

(2. continued)

(c) Suppose a point charge with positive charge Q and mass m is placed at the location labeled $r = b$ in the diagram. What is the magnitude and direction of the force exerted on the point charge due to the cylinder?
Use the coordinate system in the figure above when indicating direction.

(d) Imagine the charge Q is now released from rest at $r = b$. What is the speed of the particle when it reaches a new position $r = 3b$?

3. A circuit is formed of an ideal battery of emf $\mathcal{E} = 20\ \mathrm{V}$, an ideal inductor of inductance $L = 30\ \mathrm{mH}$, a switch S, and a circular conducting ring of radius $a = 1\ \mathrm{cm}$, as shown. The ring is connected to the circuit so that the angle in the diagram is $\theta = 120°$. All components of the circuit are ideal, except the ring which has a uniform resistance per unit length of $R/\ell = 1500/\pi\ \Omega\mathrm{m}^{-1}$. The switch S is closed at time $t = 0$.



In answering the questions below, you may ignore all inductances other than that of the inductor L.

(a) What current flows through the battery a long time after S is closed?

(b) At what time is the current equal to one-fourth of this eventual value?

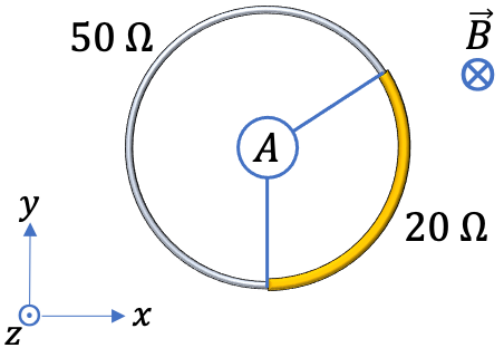
(3. continued)

(c) What fraction of the total current flows through the $120°$ arc of the ring?

(d) Use the Biot-Savart Law to find the magnitude of the B-field at the center of the ring a long time after S is closed.

(e) At the time that you found in part (b), what is the value of $\oint \vec{B} \cdot d\vec{s}$ around the path indicated in the figure above?

4. Two circular arcs of wire are joined to make a circular ring of radius r that lies in the xy-plane, as shown in the figure below. The arc with resistance of $50\ \Omega$ makes up $2/3$ of the circumference of the ring, and the arc with resistance of $20\ \Omega$ makes up the remaining $1/3$ of the circumference. The total area enclosed by the ring is $A_{ring} = \pi r^2$. An ideal ammeter (*i.e.*, zero resistance) is placed with its leads at the junction points between two arcs of wire. The ammeter is located at the geometric center of the circular ring, and the wire leading to it are along radial lines. This setup is placed in a spatially uniform but time-varying magnetic field $\vec{B} = -\beta t\,\hat{k}$, where β has SI units $^{\mathrm{T}}/_{s}$.
Assume that the wire leading to the ammeter along the radial lines are ideal.



Express your final answers to the questions in terms of the symbols r, β, t, C, μ_0, π, and real numbers, as needed.

(a) While the magnetic field is changing with time, what is the induced current I_1 in the $20\ \Omega$ arc?

(b) While the magnetic field is changing with time, what is the induced current I_2 in the $50\ \Omega$ arc?

(4. continued)

(c) While the magnetic field is changing with time, what is the current measured by the ammeter?

(d) Suppose you were to replace the ammeter with an ideal capacitor with capacitance C. While the magnetic field is changing in time, what is the maximum charge on the capacitor?

Extra Work Space (Clearly indicate which problem your work corresponds to):

Extra Work Space (Clearly indicate which problem your work corresponds to):