

[illegible]

Potentially Useful Constants and Integrals:

$$g = 9.8 \frac{\text{m}}{\text{s}^2}$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$$

$$m_e = 9.109 \times 10^{-31} \text{ kg}$$

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

$$m_p = 1.673 \times 10^{-27} \text{ kg}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad (\text{for } n \neq -1)$$

$$\int \frac{dx}{(a-x)^2} = \frac{1}{a-x}$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left(x + \sqrt{x^2 \pm a^2} \right)$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax$$

$$\int \frac{x dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \cos ax dx = \frac{1}{a} \sin ax$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \frac{x dx}{(x^2 \pm a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$$

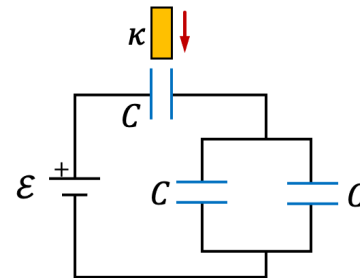
$$\int \frac{dx}{a-x} = -\ln(a-x)$$

1. A parallel plate capacitor of capacitance C is connected to a battery of voltage ΔV until it is fully charged, resulting in a potential energy U stored between the plates. If the voltage across the capacitor is doubled, the new potential energy stored between the plates is _____.

A. $8U$
 B. $4U$
 C. $2U$
 D. $\sqrt{2}U$
 E. It is not changed.

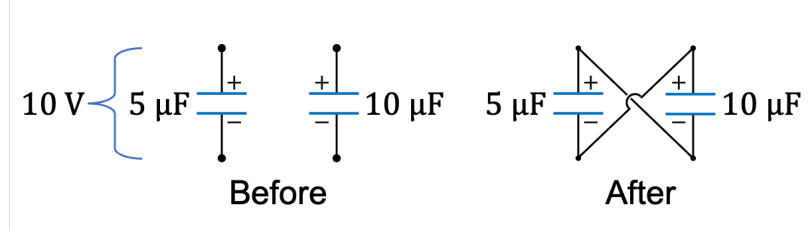
2. Consider three identical initially uncharged parallel plate capacitors, each of capacitance C , that are connected as shown in the circuit to a source of emf $\mathcal{E}$ and allowed to come to equilibrium. A dielectric with dielectric constant $\kappa = 3$ is now inserted between the plates of the upper capacitor such that it completely fills the capacitor. What is the ratio of the final equivalent capacitance (after the dielectric is inserted) to the initial equivalent capacitance? (e.g., find $C_{eq,final}/C_{eq,initial}$)

A. $3/7$
 B. $5/9$
 C. 1
 D. $9/5$
 E. $7/3$



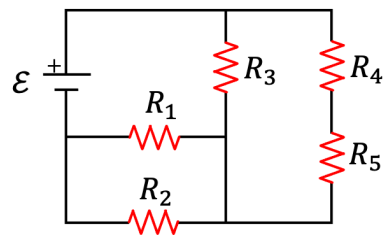
3. Consider two capacitors $C_1 = 5 \mu\text{F}$ and $C_2 = 10 \mu\text{F}$ that are each charged to a potential difference of 10 V and isolated. The positive end of C_1 is then attached to the negative end of C_2 , and the negative end of C_1 to the positive end of C_2 . What is the final charge on the capacitor C_1 after the system reaches equilibrium?

- A. $16.6 \mu\text{C}$
- B. $33.3 \mu\text{C}$
- C. $50.0 \mu\text{C}$
- D. $66.6 \mu\text{C}$
- E. $100 \mu\text{C}$



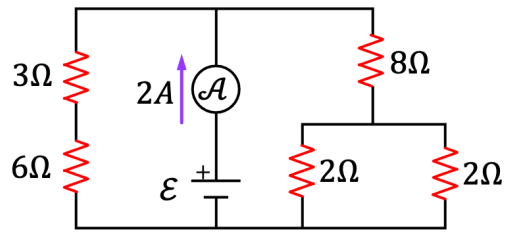
4. Consider the circuit shown below, where all the resistors, R_1 to R_5 , are identical and where the battery is ideal. For which resistor(s) is the potential difference largest?

- A. Resistors R_1 and R_2
- B. Resistors R_4 and R_5
- C. Resistors R_1 and R_5
- D. Resistor R_3
- E. Resistors R_1 , R_2 , and R_3



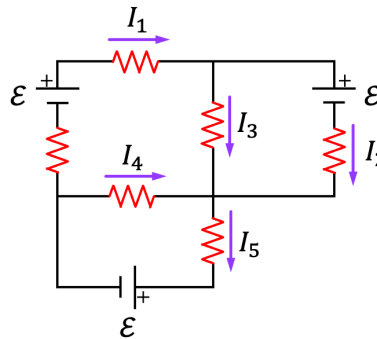
5. Consider the circuit shown, consisting of resistors, an ideal battery $\mathcal{E}$, and an ideal ammeter $\mathcal{A}$. If the current through the ammeter is 2 A, what total power dissipated in the resistors?

- A. 2.7 W
B. 5.6 W
C. 9.0 W
D. 14.3 W
E. 18.0 W



6. Consider the circuit shown where all the resistors are equal and all the batteries are ideal and provide the same emf $\mathcal{E}$. Assuming that the currents in the various branches of the circuit are directed as shown, how many of the boxed equations are correct?

- A. 1 equation
B. 2 equations
C. 3 equations
D. 4 equations
E. 5 equations

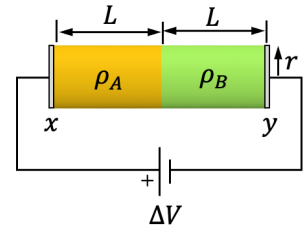


Possible Equations

$I_1 = I_2 + I_3$	$0 = \mathcal{E} + I_1 R - I_3 R + I_4 R$
$I_2 + I_4 = I_5$	$0 = -\mathcal{E} - I_2 R + I_3 R$
	$0 = \mathcal{E} + I_5 R + I_4 R$

7. Consider two cylindrical wires, A and B , that are of equal radius r and equal length L , that are connected in series, as shown. A potential difference ΔV is maintained between the ends x and y of the combination. If the resistivity of the material making up wire A is one-half that of the material making up wire B (i.e., that $\rho_A = \rho_B/2$), we can conclude that _____.

- A. the current density in wire B is twice that in wire A .
- B. the power dissipated in wire B is four times that in wire A .
- C. the potential difference across wire B is three times that across wire A .
- D. the electron drift velocities are the same in both wires.
- E. None of the above.



8. Consider a charged particle moving in a uniform time-independent magnetic field. Which of the following statements must be true?
- (I) The velocity of the particle is constant.
 - (II) The kinetic energy of the particle is constant.
 - (III) The angular momentum of the particle about an arbitrary point is constant.
- A. Statement (I) only.
 - B. Statement (II) only.
 - C. Statement (III) only.
 - D. Statements (I) and (II) only.
 - E. Statements (II) and (III) only.

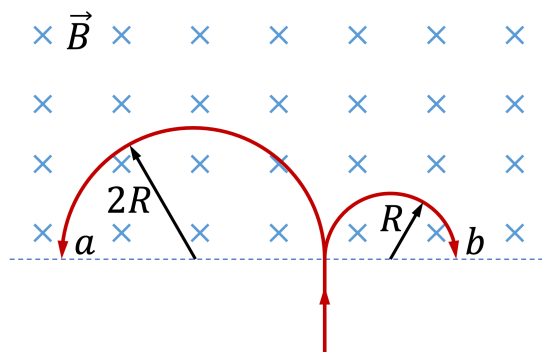
9. An electron with velocity $\vec{v} = v_0 \hat{k}$ is moving in the presence of a uniform magnetic field $\vec{B} = B_0 \hat{j}$. If the electron continues undeflected, we can conclude that an electric field must be present of the form _____. (Note: v_0 , B_0 , and E_0 are all magnitudes.)

- A. $\vec{E} = +E_0 \hat{i}$
- B. $\vec{E} = -E_0 \hat{i}$
- C. $\vec{E} = +E_0 \hat{k}$
- D. $\vec{E} = -E_0 \hat{k}$
- E. $\vec{E} = -E_0 \hat{j}$

10. Two particles, a and b , with charges of the same magnitude enter a uniform magnetic field $\vec{B}$ directed into the page. The particles enter the magnetic field with the same speed v_0 and travel perpendicular to the magnetic field along the trajectories shown. Which of the following statements must be true?

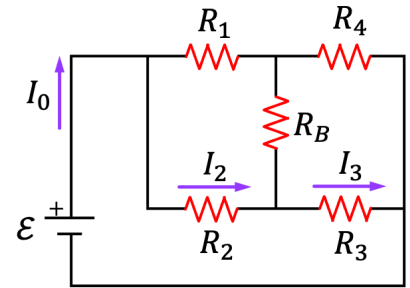
- (I) particle b has twice the mass of particle a .
- (II) particles a and b have equal cyclotron frequencies.
- (III) particle a has a positive charge.

- A. Statement (I) only.
- B. Statement (II) only.
- C. Statement (III) only.
- D. Statements (I) and (II) only.
- E. Statements (II) and (III) only.



Free Response Questions: (20 points each)

1) In the circuit shown, an ideal battery is connected to an array of five resistors, four of which have known resistance. The known resistances are $R_1 = 10\ \Omega$, $R_2 = 50\ \Omega$, $R_3 = 40\ \Omega$, and $R_B = 10\ \Omega$. Resistor R_4 has a variable resistance which is set so that the labeled currents are related to one another as $I_0 = 5I_2$ and $I_3 = 2I_2$. The ideal battery provides a voltage $\mathcal{E} = 13\text{ V}$ to the resistor array.



(a) What is the value of the current I_2 ?

(b) What is the current in the bridging resistor R_B ? Report both the value and the direction of the current.

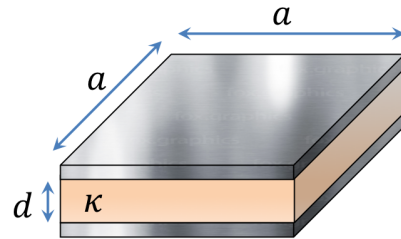
1) (continued):

(c) What is the equivalent resistance of this circuit?

(d) To what value of R_4 should you set the variable resistor so that there is no current through the bridging resistor R_B ?

2) In this course you have seen how a non-ideal battery is modeled as an ideal battery in series with a resistor. A non-ideal capacitor can be modeled as an ideal capacitor with an internal resistance between the two plates, which explains how the capacitor will discharge by “leaking” charge through the internal resistance, even when the capacitor is completely isolated.

The non-ideal capacitor shown consists of a pair of parallel square plates with side length a that are at distance d apart. The capacitor is filled with a dielectric with dielectric constant κ . At time $t = 0$, the charge on the capacitor is Q and the capacitor is isolated from the power source used to charge it up. During a time interval Δt , the capacitor has discharged to one half of its initial charge.



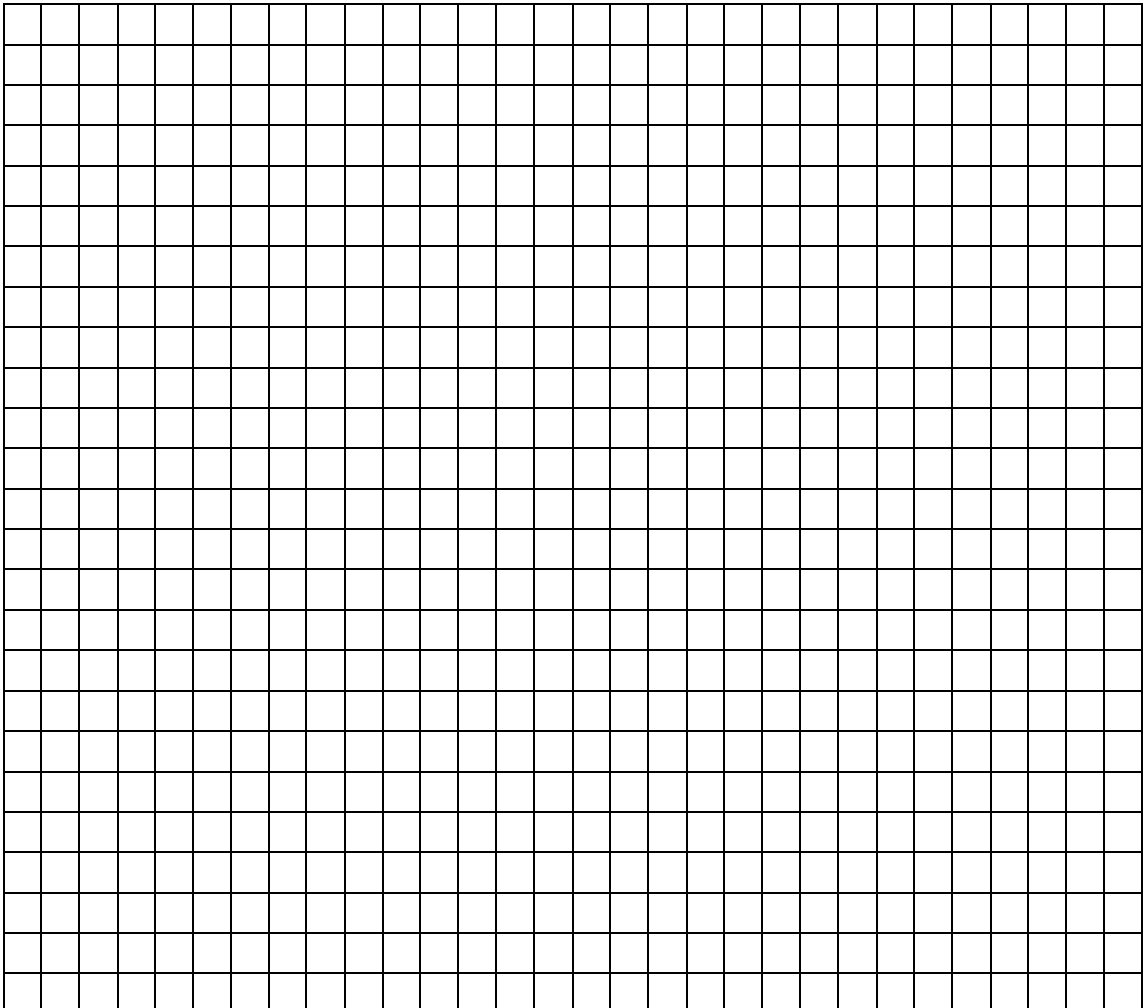
(a) Sketch the equivalent circuit for the non-ideal capacitor.

(b) What is the time constant, τ , for the discharging non-ideal capacitor? Express your answer **only** in terms of Δt , and any numerical constants, as needed.

2) (continued):

(c) What is the resistivity, ρ , of the dielectric? Express your answer **only** in terms of $a, d, Q, \Delta t, \kappa, \epsilon_0$, and any numerical constants, as needed.

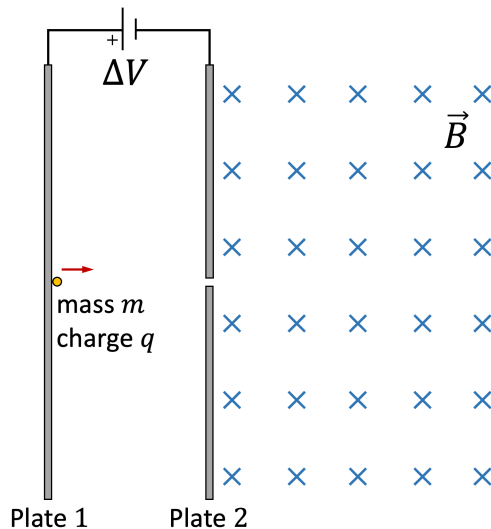
(d) Sketch the voltage across the capacitor as a function of time from $t = 0$ to $t \gg \Delta t$.



3) A potential difference ΔV is set up between two plates labeled as Plate 1 and Plate 2. A point particle of mass m and charge $q > 0$ is placed at rest at Plate 1 as shown in the figure below. The positively-charged particle is released from rest and then accelerates across the space between the plates.

At $t = 0$, the particle passes through a small opening in Plate 2 and enters a region in which there is a uniform magnetic field of magnitude B that is directed into the page as shown. In this problem, you should ignore the effects due to gravity.

Important: Report your answers to parts (a) and (c) in terms of ΔV , m , q , B , and real numbers, as needed.



- (a) What is the speed of the particle as it passes through the small opening at $t = 0$?
- (b) On the diagram above, sketch the path that the particle follows while it is in the region of uniform magnetic field.
- (c) How long in time does it take the particle to strike Plate 2 after $t = 0$?

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- (c) How long in time does it take the particle to strike Plate 2 after $t = 0$?

3) (continued):

An experiment is performed in which identical point particles with mass m and charge q are accelerated across the space between the plates and pass through the small opening. The potential difference, ΔV , between the plates and the magnitude of the magnetic field B are varied to cause the particles to always strike Plate 2 at a distance of $r = 0.5$ m from the small opening. The values of ΔV and B used are shown in the table below.

ΔV : Potential difference	60 V	100 V	140 V
B : Magnetic field B	0.00524 T	0.00675 T	0.00798 T

Important: Report your answer to part (d) in terms of ΔV , m , q , B , and real numbers, as needed.

- (d) Suppose you were to plot ΔV on the horizontal (“x”) axis, what quantity should you plot on the vertical (“y”) axis so that the line of best fit has a slope equal to the mass-to-charge ratio $\left(\frac{m}{q}\right)$ of the particle?

- (e) What is the approximate numerical value of the mass-to-charge ratio $\left(\frac{m}{q}\right)$ of the particle?

Extra Work Space (Clearly indicate which problem your work corresponds to):