Phys 102 Midterm 2 Spring 2023

Solution

Multiple-Choice

1). $U_i = \frac{1}{2} C \Delta V^2$, $U_f = \frac{1}{2} C [2\Delta V]^2 = 4 U_i$

$\Rightarrow \boxed{B}$

2). $[C_{eq}]_i = \left[\frac{1}{C} + \frac{1}{2C}\right]^{-1} = \frac{2C}{3}$ $\boxed{k=3}$

$[C_{eq}]_f = \left[\frac{1}{kC} + \frac{1}{2C}\right]^{-1} = \frac{2kC}{2+k} = \frac{6}{5} C$

$\Rightarrow \frac{(C_{eq})_f}{(C_{eq})_i} = \frac{6}{5} \frac{3}{2} = \frac{9}{5} \Rightarrow \boxed{D}$

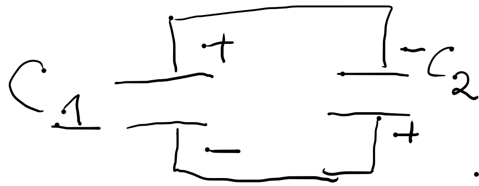
3).

Before

$(Q_1)_i = C_1 V$; $(Q_2)_i = C_2 V$

$$\boxed{\begin{aligned} C_1 &= 5\,\mu F \\ C_2 &= 10\,\mu F \end{aligned}}$$

After



By the conservation of charge

$$(Q_1)_f + (Q_2)_f = (Q_2)_i - (Q_1)_i$$

$$\Downarrow$$

$$(\Delta V_1)_f = (\Delta V_2)_f$$

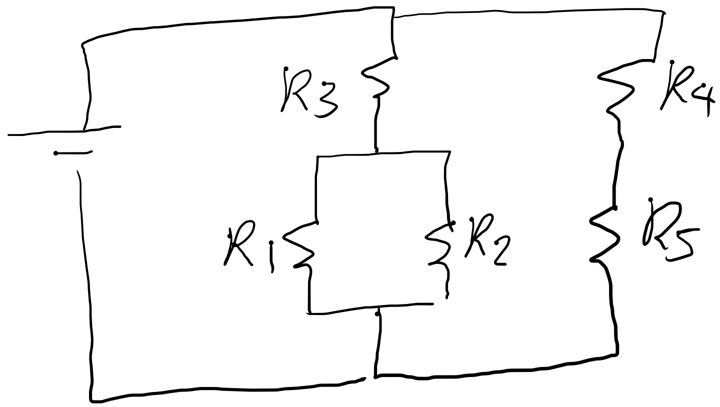
$$\Rightarrow \frac{(Q_1)_f}{C_1} = \frac{(Q_2)_f}{C_2} \Rightarrow (Q_1)_f + \frac{C_2}{C_1}(Q_1)_f = (C_2 - C_1) V$$

$$\Rightarrow (Q_1)_f = \frac{(C_2 - C_1) V}{\left[1 + \frac{C_2}{C_1}\right]}$$

$$= 16.67\,\mu C$$

$$\Rightarrow \boxed{A}$$

4).



$$R_1 = R_2 = R_3 = R_4 = R_5 = R$$

$$\Downarrow$$

$$\Rightarrow \quad \Delta V_4 = \Delta V_5 = \frac{\varepsilon}{2}$$

$$\Rightarrow \quad \Delta V_3 = \left[\frac{\varepsilon}{R + \frac{R}{2}}\right] R$$

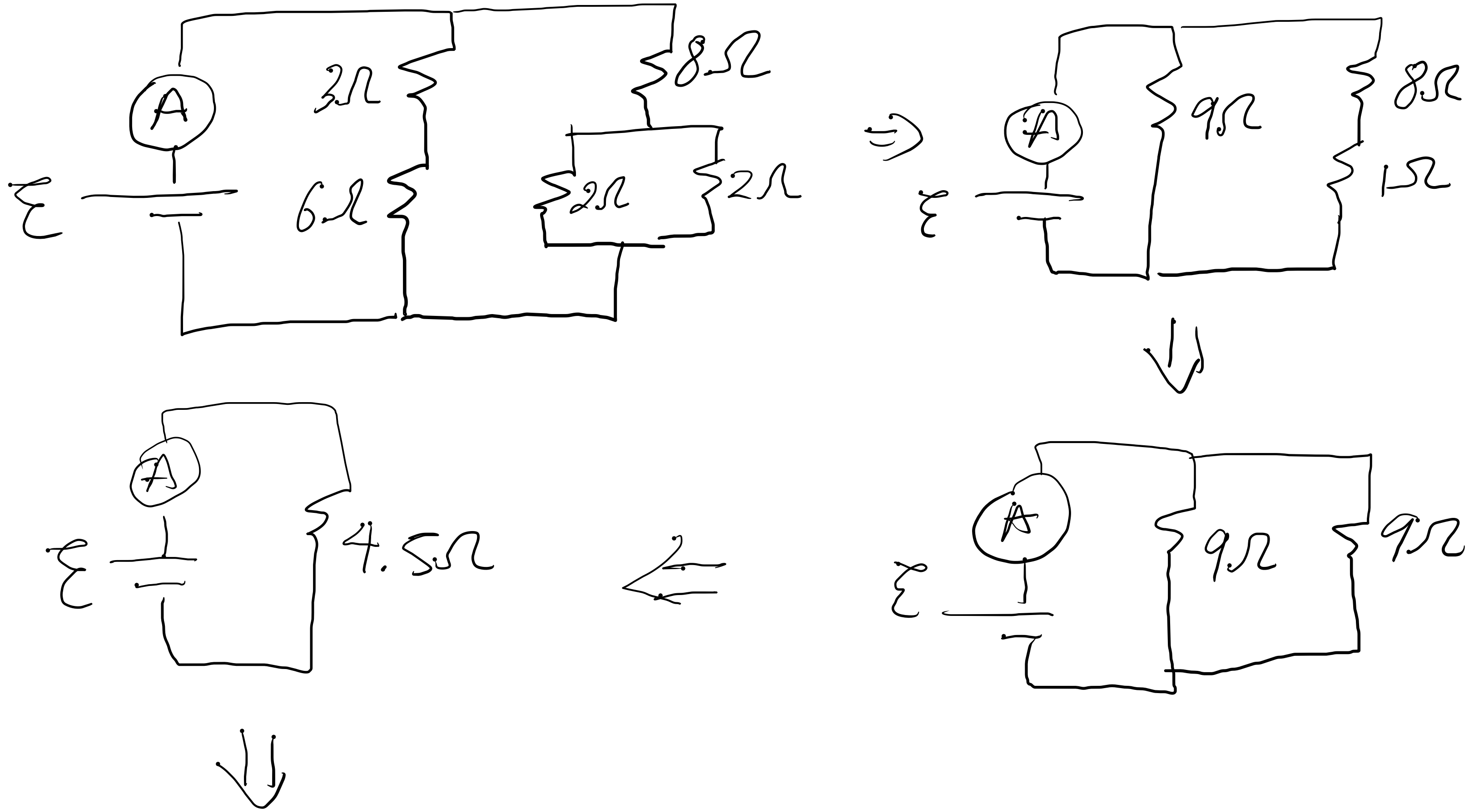
$$= \frac{2}{3}\varepsilon$$

$$\Rightarrow \quad \Delta V_1 = \Delta V_2 = \frac{\varepsilon}{3}$$

$$\Rightarrow \quad \Delta V_3 > (\Delta V_4 = \Delta V_5) > (\Delta V_1 = \Delta V_2)$$

$$\Rightarrow \quad \boxed{D}$$

5).



$$\text{Power} = I^2 R_{eq} = (2A)^2 (4.5\Omega)$$

$$\Rightarrow \quad \text{Power} = 18\,W \Rightarrow \boxed{E}$$

6).

$I_1 = I_2 + I_3$ ✓

$I_5 = I_2 + I_3 + I_4$

$I_5 = I_1 + I_4$

$\mathcal{E} - 2I_1R - I_3R + I_4R = 0$

$-\mathcal{E} - I_2R + I_3R = 0$ ✓

$\mathcal{E} + I_5R + I_4R = 0$ ✓

$\Rightarrow \boxed{C}$

7).

$\boxed{I_A = I_B} \Rightarrow \boxed{J_A = J_B}$

$\boxed{R_A = \frac{R_B}{2}} \Rightarrow \boxed{\Delta V_A = \frac{\Delta V_B}{2}}$ ✓

$\text{Power} = I^2R \Rightarrow \boxed{(\text{Power})_A = \frac{(\text{Power})_B}{2}}$

$I = qnAv_d$

$(qn)_A \neq (qn)_B \Rightarrow (v_d)_A \neq (v_d)_B$

$\Rightarrow \boxed{E}$

8). $\vec{F}_B = q\vec{v} \times \vec{B}$

Here, $|\vec{v}|$ is constant, but not $\vec{v}$

$\frac{1}{2} m |\vec{v}|^2$ is constant.

Since the magnetic force could exert a torque on a particle and

$$\frac{d\vec{L}}{dt} = \vec{\tau},$$

$\vec{L}$ does not need to be constant.

$\Rightarrow \boxed{B}$

9).

$$\vec{F} = -e\vec{v} \times \vec{B} + -e\vec{E} = 0$$

$$\Rightarrow -ev_0\hat{k} \times B_0\hat{j} - e\vec{E} = 0$$

$$\Rightarrow e\vec{E} = -eV_0B_0(\hat{k} \times \hat{j})$$

$$= -ev_0 B_0(-\hat{i})$$

$$\Rightarrow \vec{E} = v_0 B_0 \hat{i} = +E_0\hat{i} \Rightarrow \boxed{A}$$

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(10). Here, $qvB = \frac{mv^2}{r} \Rightarrow r = \frac{m}{q}\frac{v}{B}$

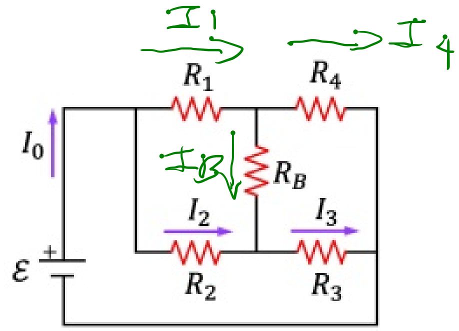
$$\Rightarrow \left(\frac{m}{q}\right)_a = 2\left(\frac{m}{q}\right)_b$$

By right-hand rule, $q_a > 0 \Rightarrow \boxed{C}$

$$\omega = \frac{v}{r} = B\frac{q}{m} \Rightarrow \omega_b = 2\omega_a$$

Free Response

1). a).



$$\Rightarrow \quad \mathcal{E} - I_2 R_2 - I_3 R_3 = 0$$

$$\Rightarrow \quad \mathcal{E} - I_2 R_2 - (2 I_2) R_3 = 0$$

$$\Rightarrow \quad \mathcal{E} = I_2 \left[R_2 + 2R_3\right]$$

$$\Rightarrow \quad \boxed{I_2 = \frac{\mathcal{E}}{\left[R_2 + 2R_3\right]} = \frac{13\,V}{\left[50\,\Omega + 80\,\Omega\right]} = 0.1\,A}$$

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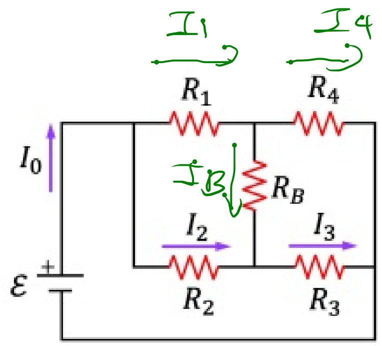
b).

$$I_3 = I_2 + I_B \quad \Rightarrow \quad I_B = I_3 - I_2$$

$$= (2 I_2) - I_2 = I_2$$

$$\Rightarrow \quad \boxed{I_B = I_2 = 0.1\,A} \quad \boxed{\text{from top to bottom}}$$

1). c).



$$R_{eq} = \frac{\varepsilon}{I_0} = \frac{\varepsilon}{(5I_2)}$$

$$\Rightarrow \boxed{R_{eq} = \frac{13V}{(5(0.1A))} = 26\Omega}$$

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d). If $I_B = 0$, then $I_1 = I_4$ & $I_2 = I_3$

$$\Rightarrow \quad I_1(R_1 + R_4) = I_2(R_2 + R_3) \Rightarrow \frac{I_2}{I_1} = \frac{(R_1+R_4)}{(R_2+R_3)}$$

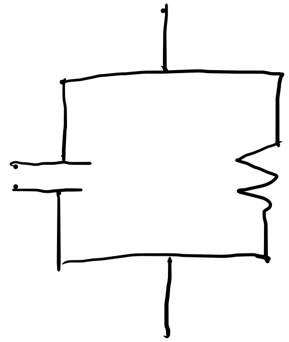
Also, $I_1 R_1 = I_2 R_2 \Rightarrow \frac{I_2}{I_1} = \frac{R_1}{R_2}$

$$\Rightarrow \frac{(R_1+R_4)}{(R_2+R_3)} = \frac{R_1}{R_2} \Rightarrow \left(\frac{R_1}{R_2}\right)(R_2+R_3) - R_1 = R_4$$

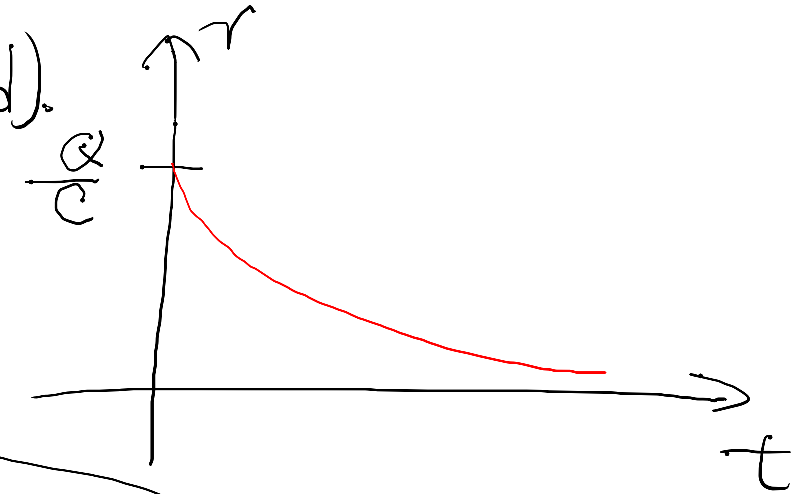
$$\Rightarrow \boxed{R_4 = R_1 + \frac{R_1}{R_2}R_3 - R_1 = \frac{R_1}{R_2}R_3 = \left(\frac{10\Omega}{50\Omega}\right)(40\Omega) = 8\Omega}$$

2).

a).



d).



b).

$$\frac{Q}{2} = Q e^{-t/\tau}$$

$$\Rightarrow \ln\left(\frac{1}{2}\right) = \frac{-\Delta t}{\tau}$$

$$\Rightarrow \boxed{\tau = \frac{-\Delta t}{\ln\left(\frac{1}{2}\right)} = \frac{\Delta t}{\ln(2)}}$$

c).

$$\tau = RC \ ; \ R = \frac{\rho d}{a^2} \ ; \ C = \frac{k \varepsilon_0 a^2}{d}$$

$$\Rightarrow \tau = \frac{\rho d}{a^2} \frac{k \varepsilon_0 a^2}{d} = \rho k \varepsilon_0 \Rightarrow \rho = \frac{\tau}{k \varepsilon_0}$$

$$\Rightarrow \boxed{\rho = \frac{-\Delta t}{k \varepsilon_0 \ln\left(\frac{1}{2}\right)} = \frac{\Delta t}{k \varepsilon_0 \ln(2)}}$$

3).

a). $q\Delta V = \frac{1}{2}mv^2$

$$\Rightarrow \boxed{v = \sqrt{\frac{2q\Delta V}{m}}}$$

b).

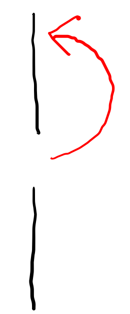


plate 2

c). $qvB = \frac{mv^2}{r} \Rightarrow r = \frac{m}{q}\frac{v}{B}$

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

$$\boxed{\Delta t = \frac{T}{2} = \frac{\pi m}{qB}}$$

d). $r = \frac{m}{q}\frac{v}{B}$; $v = \sqrt{\frac{2q\Delta V}{m}} \Rightarrow r = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}}$

$$\Rightarrow r^2 = \frac{m^2}{q^2B^2}\frac{2q\Delta V}{m} \Rightarrow r^2 = \frac{2\Delta V}{B^2}\frac{m}{q}$$

$$\Rightarrow \frac{B^2r^2}{2} = \frac{m}{q}(\Delta V) \Rightarrow$$ Plot $\frac{B^2(0.25\,m)^2}{2}$ on the vertical axis

e). $\frac{m}{q} = \frac{(Br)^2}{2\Delta V}$

$$= \frac{(0.00524\,T(0.25\,m))^2}{2(60\,V)}$$

$$\boxed{= 1.4 \times 10^{-8}\ kg/C}$$