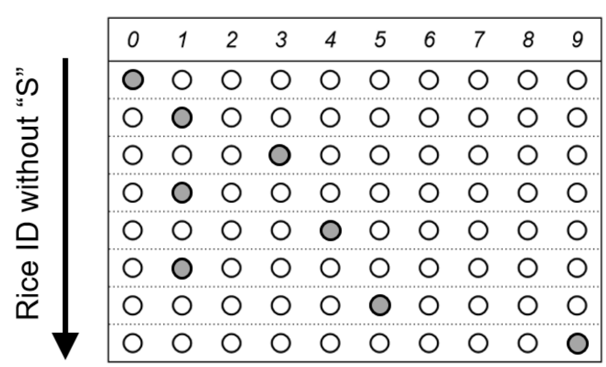
Physics 102 Spring 2023 Exam 1

Time allowed: 90 minutes, closed book

Instructions:
Please print your name and NetID in **two** places: on the top of this cover sheet, and on the multiple-choice answer sheet. If you use any additional paper for your work, please make sure to print your name on the top of each sheet and staple the extra sheets to this exam packet when you hand in your work at the end of the session. **(No name, No credit!)**

- There are 10 multiple choice questions and 3 free response questions in total.
- The maximum possible points are 100 points.
- Mark your answers to the multiple-choice questions on the answer sheet provided. Make sure to fill the appropriate bubble **completely** using a #2 pencil, or a black pen. Any multiple-choice responses written on pages other than the answer sheet will NOT be graded.
- Write all your solutions to the free response questions in the space provided in the exam packet, or on the extra space provided at the end of the exam packet. Make sure that it is very clear which problem your work corresponds to. If needed, extra paper will be provided at the front of the exam room. Remember to print your name on any extra sheets and staple them to the exam packet.
- When you finish, please place the exam packets and the multiple-choice answer sheet in two separate piles at the front of the exam room. If you used additional sheets of paper, make sure to staple them to the exam packet. Hand in all your work at the end of the 90-minute exam period.
- You are not allowed to take anything written away from the exam room.
- You may not use phones, computers, tablets, or any other web connected device during the exam.
- You may not use the symbolic manipulation or graphing capabilities of your calculator. (You can look up trig functions, *i.e.*, calculating sin(45) is not a symbolic manipulation.)
- You may not store, or use pre-stored formulae, saved in your calculator's memory, or anything written down in advance of entering the exam.

On your multiple-choice answer sheet, you will need to fill in your Rice ID. Beginning with the numbers “01”, enter your Rice ID by bubbling in one number per row. In the example below, the Rice ID entered is "S01314159".



Potentially Useful Constants and Integrals:

$$g = 9.8\ \frac{\text{m}}{\text{s}^2}$$

$$e = 1.602 \times 10^{-19}\ \text{C}$$

$$\varepsilon_0 = 8.854 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$$

$$m_e = 9.109 \times 10^{-31}\ \text{kg}$$

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$$

$$m_P = 1.673 \times 10^{-27}\ \text{kg}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (\text{for } n \neq -1)$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{a-x} = -\ln(a-x)$$

$$\int \frac{dx}{(a-x)^2} = \frac{1}{a-x}$$

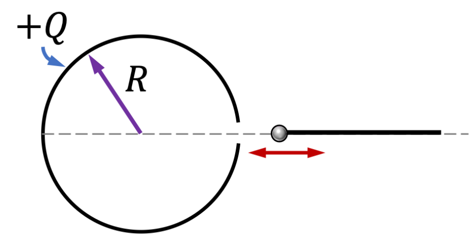
$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{(x^2 \pm a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$$

1. Consider a spherical conducting shell of radius R that carries a large positive charge, $+Q$, and that has a small hole cut in it. A small, initially uncharged, conductor mounted on an insulating rod is passed through the hole, touched to the opposite inner face and withdrawn, all without touching the edges of the hole. Following this, the small conductor has:



   A. no charge.

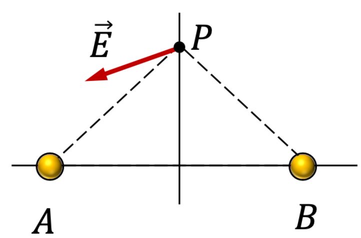
   B. a small positive charge.

   C. a small negative charge.

   D. a large positive charge.

   E. a large negative charge.

2. The direction of the electric field at point P equidistant from two charged bodies A and B is given by the vector $\vec{E}$ shown in the figure. From this we can conclude that:



   A. both charges are negative.

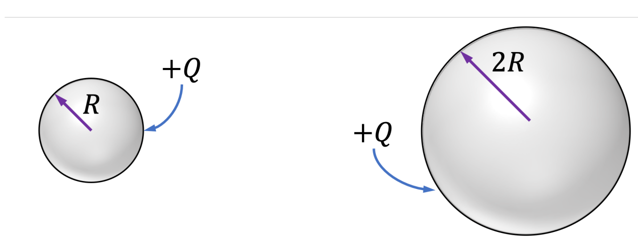
   B. both charges are positive.

   C. charge A is positive, B negative.

   D. charge A is negative, B positive.

   E. more information is needed to deduce their charges.

3. Consider two isolated, well-separated, spherical conductors of radii R and $2R$ that have equal initial charges $+Q$. If they are briefly connected by a thin insulated wire that is then removed, what is the new charge on the sphere of radius R?



   A. $4Q/3$

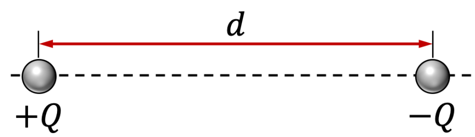
   B. $2Q/3$

   C. Q

   D. $3Q/2$

   E. $3Q/4$

4. Consider two small identical spherical conductors of radius r that are separated by some distance $d \gg r$. The spheres initially carry equal but opposite charges $+Q$ and $-Q$, as shown, and the magnitude of the force between them is F_0. If half the charge on one sphere is moved to the other, the magnitude of the force between them becomes:



A. $F_0/4$

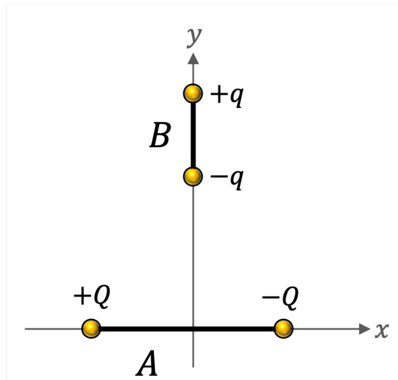
B. $F_0/2$

C. $2F_0/3$

D. $3F_0/4$

D. F_0

5. Consider the electric dipoles A (with charges $\pm Q$) and B (with charges $\pm q$) positioned, as shown below. Which of the following statements is true?



A. Dipole B experiences a net force but no torque.

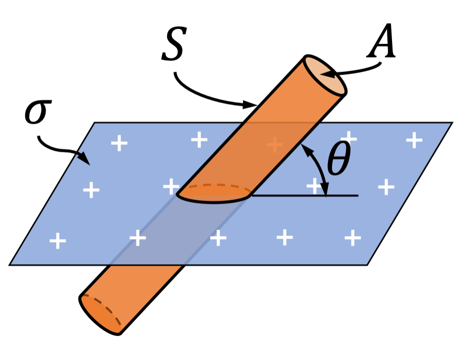
B. Dipole B experiences a torque but no net force.

C. Dipole B experiences no net force or torque.

D. Dipole B experiences a net force and a torque.

E. This cannot be determined without specifying the magnitude of the charges q and Q.

6. A closed cylindrical surface S passes through a planar sheet of charge with uniform charge density σ, as shown below. The cylinder has cross-sectional area A and makes an angle θ with the planar sheet. The electric flux through the closed cylindrical surface is:



A. $\sigma A/\varepsilon_0$

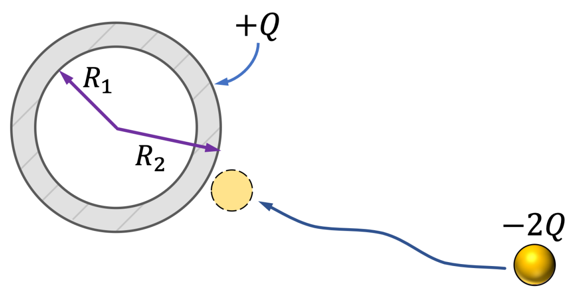
B. $\sigma A \sin\theta / \varepsilon_0$

C. $\sigma A \cos\theta / \varepsilon_0$

D. $\sigma A/(\varepsilon_0 \sin\theta)$

E. $\sigma A/(\varepsilon_0 \cos\theta)$

7. Consider a spherical conductor with inner radius R_1 and outer radius R_2 that initially carries a charge $+Q$. If a second charge $-2Q$ is brought up just outside, but not touching the spherical conductor, we can conclude that the inner surface of the conductor will have:



A. a net charge of $+3Q$

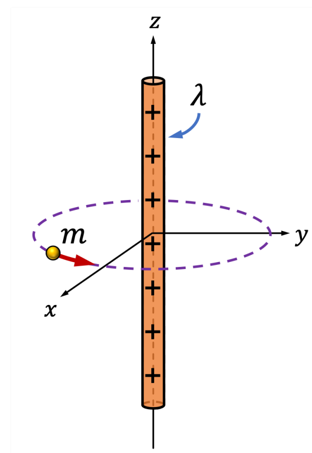
B. a net charge of $+2Q$

C. a net charge of $+Q$

D. a net charge of $-Q$

E. a net charge of zero.

8. Consider a long plastic rod that lies along the *z*-axis and carries a uniform charge density $+\lambda$. A small object of mass m and charge $-Q$ is orbiting the rod in the *xy* plane in a circular orbit of radius r with speed v_1. If the charge on the object is increased to $-2Q$ and it is desired that it maintains the same orbit, its orbital speed v_2 must become:



A. $v_2 = v_1$

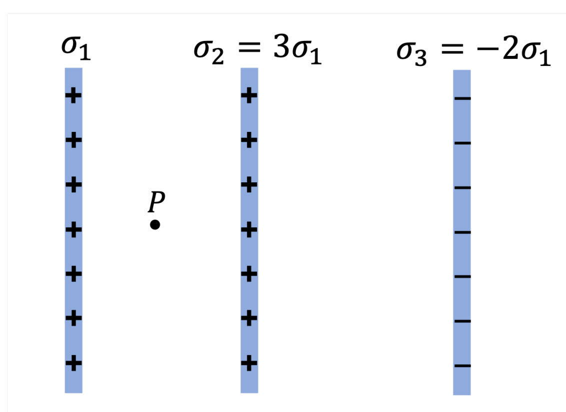
B. $v_2 = \sqrt{2}\, v_1$

C. $v_2 = v_1/\sqrt{2}$

D. $v_2 = 2v_1$

E. $v_2 = v_1/2$

9. Consider three large parallel planar sheets of charge with surface charge densities σ_1, σ_2, and σ_3, where $\sigma_2 = 3\sigma_1$ and $\sigma_3 = -2\sigma_1$. The magnitude of the electric field at the point P shown in the figure is:



A. $\sigma_1/2\varepsilon_0$

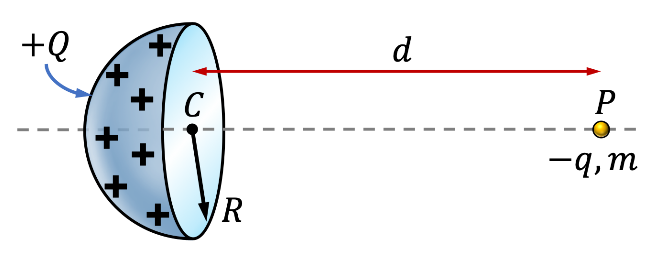
B. σ_1/ε_0

C. $2\sigma_1/\varepsilon_0$

D. $6\sigma_1/\varepsilon_0$

E. zero

10. Consider an insulating hemispherical shell of radius R that carries a charge $+Q$ uniformly distributed over its surface. A particle of mass m and charge $-q$ is released from rest at a distant point P on the axis of the shell where $d \gg R$. The speed of the particle when it arrives at the center C of the hemisphere is most nearly:



A. $\left(\frac{2k_e\, q\, Q\, d}{m\, R}\right)^{1/2}$

B. $\left(\frac{2k_e\, q\, Q}{m\, d\, R}\right)^{1/2}$

C. $\left(\frac{2k_e\, q\, Q}{m\, R^2}\right)^{1/2}$

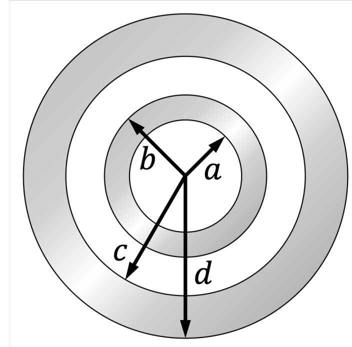
D. $\left(\frac{2k_e\, q\, Q}{m\, R}\right)^{1/2}$

E. $\left(\frac{2k_e\, q\, Q}{m\, d\, R^2}\right)^{1/2}$

Free Response Questions: (20 points each)

1. Consider two nested, initially uncharged, spherical conducting shells as shown in the figure to the right. The smaller shell has inner radius a and outer radius b. The larger shell has inner radius c and outer radius d. In this problem we define r as the distance from the center of the spherical shells, and we assume that the electric potential $V = 0$ at $r = \infty$.

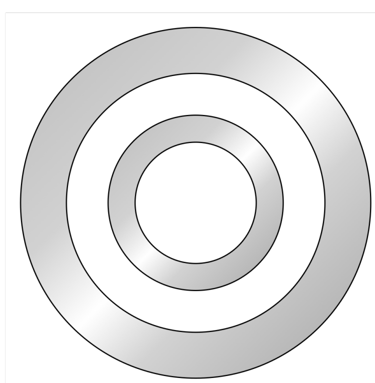
   Please report your answers in terms of $Q, \varepsilon_0, r, a, b, c, d$ and natural numbers, as needed.



For parts (a)-(c) assume both shells are NOT connected to anything, and that a positive charge $+Q$ is instantly introduced at the geometric center of the arrangement (i.e., at $r = 0$).

a) Once the system reaches equilibrium what are the charge densities for the surfaces at $r = a, r = b, r = c$, and $r = d$?

b) Once the system reaches equilibrium, sketch the electric field lines at all regions between $r = 0$ and $r = 2d$ on the cross-section figure below. Label $\vec{E} = 0$ in regions where the electric field is zero.



1) (continued):

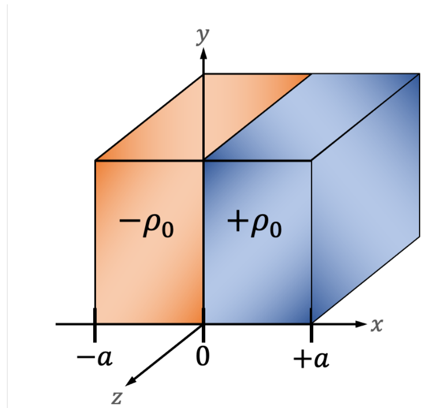
c) Once the system reaches equilibrium what is the electric potential for the region where $r > d$?

For part (d) assume that we take the system as described in parts (a)-(c) and <u>now we ground the outer shell.</u>

d) Once the system reaches a new equilibrium, what are the charge densities for the surfaces at $r = a, r = b, r = c,$ and $r = d$? Also, what is the potential difference between the two shells?

2. Consider two parallel infinite slabs of charge, both of thickness a, in direct contact with each other along the plane $x = 0$, as shown in the figure. Each slab has a uniform volume charge density as described below:

$\rho = +\rho_0$ for $0 < x < a$;
$\rho = -\rho_0$ for $-a < x < 0$;
$\rho = 0$ for $|x| > a$.
Note: The slabs are each infinite
In the y and z directions.



Report the answers for part (c) and (d) in terms of ε_0, ρ_0, a, and natural numbers, as needed. Report the direction of the electric field using the Cartesian unit vectors.

a) Where is the **minimum** electric field within the two slabs ($-a \leq x \leq a$)?
To receive credit for this part of the question, you must explain your reasoning.

b) Where is the **maximum** electric field within the two slabs ($-a \leq x \leq a$)?
To receive credit for this part of the question, you must explain your reasoning.

2) (continued):

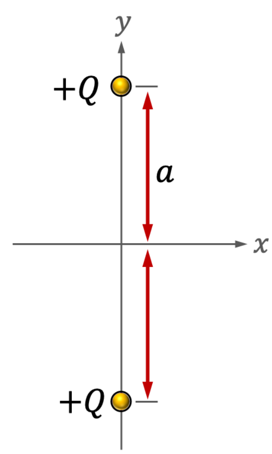
Report the answers for part (c) and (d) in terms of ε_0, ρ_0, a, and natural numbers, as needed. Also, report the direction of the electric field using the Cartesian unit vectors.

c) What is the electric field at $x = 0$? You must include the direction in your answer.

d) What is the electric field at $x = a/2$? You must include the direction in your answer.

3. Two identical charged particles of charge $+Q$ are placed on the y-axis, as shown in the figure below. Each charged particle is a distance a from the origin. For part (a), (b), and (c), assume the two charged particles are fixed in place. In this problem we will define the zero of the electric potential to be at $r = \infty$.

   Please report your answers in terms of k, Q, a, x, y and natural numbers, as needed.



   a) What is the electric potential at the coordinate (x, y), excluding the points $(0, \pm a)$?

   b) Sketch the electric potential along the x-axis, $V(x, y = 0)$ for all values of x.

3) (continued):

c) What is the electric field at the coordinate (x, y), excluding the points $(0, \pm a)$? Report the direction of the electric field using the Cartesian unit vectors.

d) If both charges are released and allowed to move freely, what is the sum of their kinetic energies when they are a distance $5a$ from each other?

Extra Work Space (Clearly indicate which problem your work corresponds to):