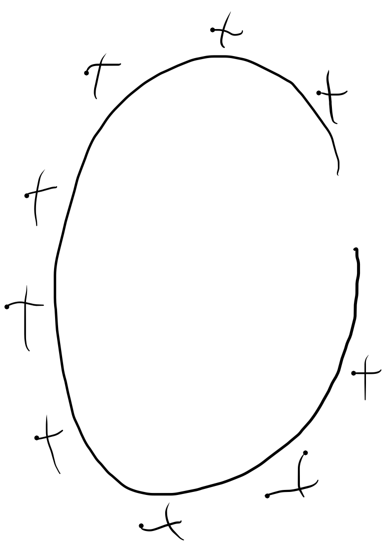
Phys 102 Sp23 Midterm 1 Solution

1).

There are zero charge on the inner surface of the sphere.

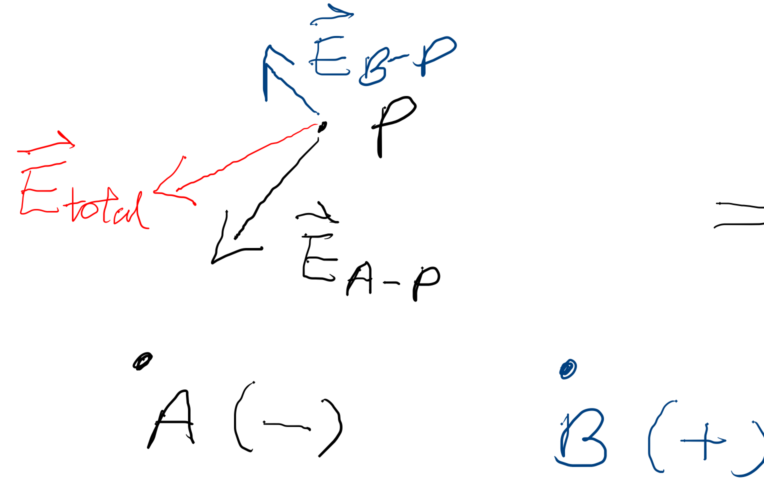


So, the small conductor did not come into contact with any charge

$\Rightarrow \boxed{A}$

This is true in the limit that the hole is infinitesimally small

2).



$\Rightarrow \boxed{D}$

3). After the connection

$$V_L = V_R$$

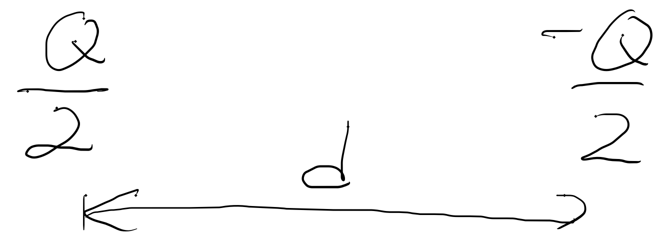
$$\Rightarrow \frac{kQ_L}{R} = \frac{kQ_R}{2R} \Rightarrow Q_R = 2Q_L$$

By charge conservation

$$Q_L + Q_R = 2Q \Rightarrow 3Q_L = 2Q$$

$$\Rightarrow Q_L = \frac{2}{3}Q \Rightarrow \boxed{B}$$

4). $|\vec{F}_0| = \frac{-kQ^2}{d^2}$, After the transfer of charge

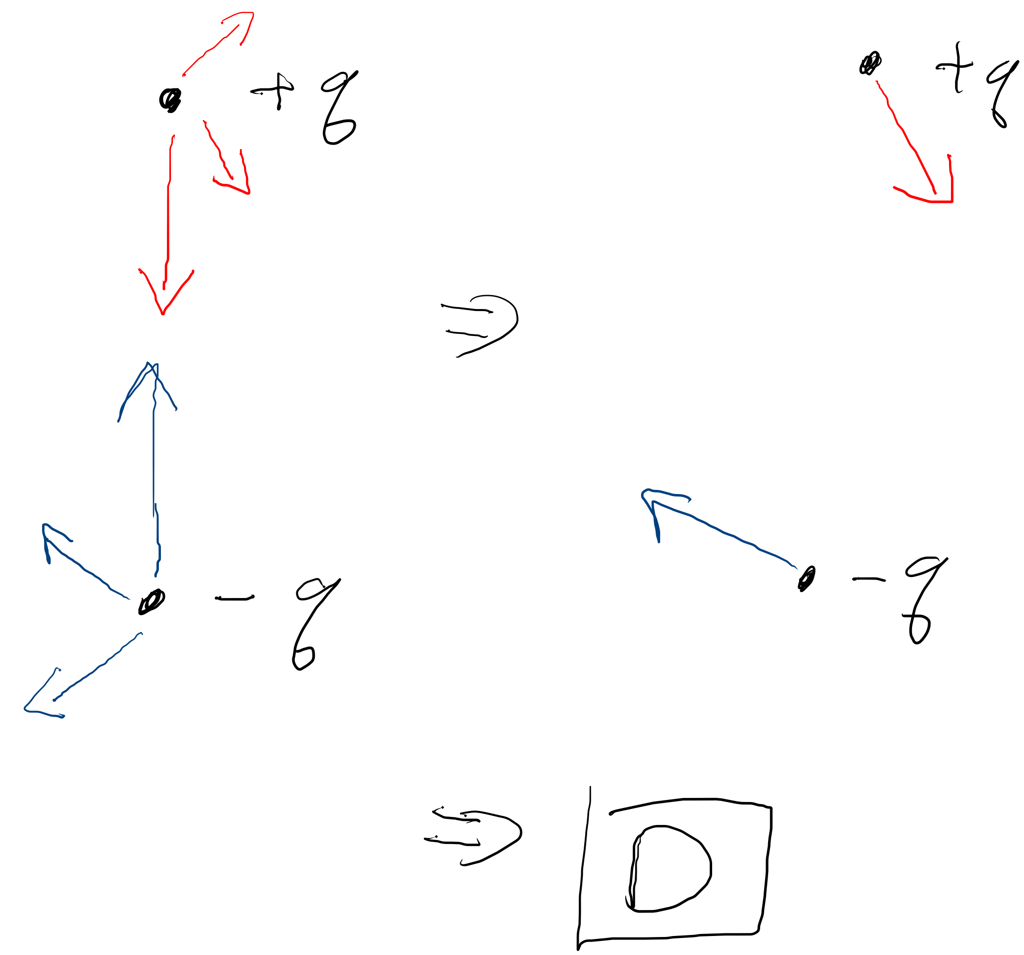


$$\Rightarrow |\vec{F}_f| = \frac{-k\left(\frac{Q}{2}\right)^2}{d^2}$$

$$\Rightarrow |\vec{F}_f| = \frac{-kQ^2}{d^2}\,\frac{1}{4} \Rightarrow F_f = \frac{F_0}{4}$$

$$\Rightarrow \boxed{A}$$

5).



6). $\Phi = \oint \vec{E} \cdot dA = \dfrac{\sigma A}{\varepsilon_0 \sin\theta}$

Here, If $\theta = 90^\circ$, $\Phi = \dfrac{\sigma A}{\varepsilon_0}$

If $\theta = 0^\circ$, $\Phi \rightarrow \infty$

$\Rightarrow \boxed{D}$

This is correct as long as the angle is not zero.

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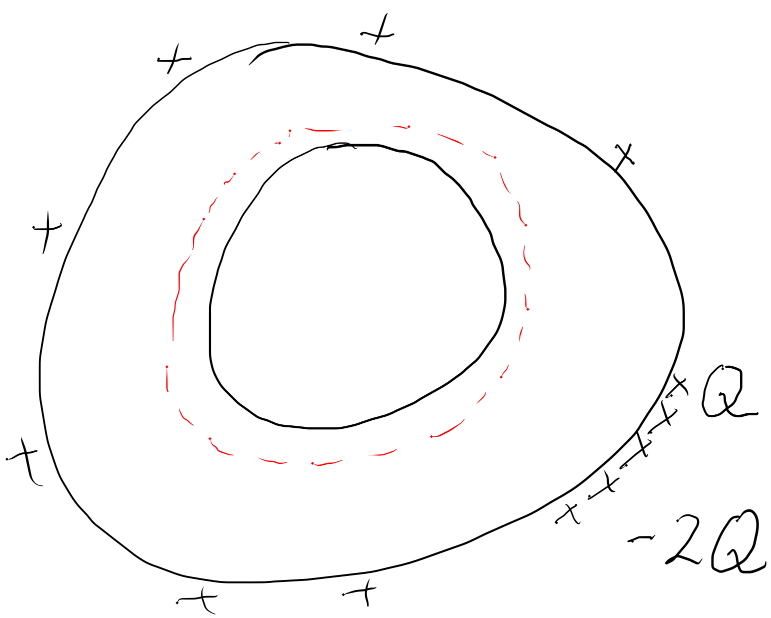
7).



No $\vec{E}$-field inside the conductor

so, no charge on the inner surface.

$\Rightarrow \boxed{E}$

8). Here,

$$a_r \propto F \propto \frac{\lambda Q}{r}$$

$$\Rightarrow \frac{v_1^2}{r} \propto \frac{\lambda Q}{r} \Rightarrow v_1 \propto (\lambda Q)^{1/2}$$

If we doubled the charge, then the speed must increase by a factor $\sqrt{2}$

$$\Rightarrow \boxed{B}$$

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9). Recall, the $\vec{E}$-field from an infinite plane of charge is constant in magnitude.

So, at P

$$\frac{\sigma_2}{2\varepsilon_0}\hat{\imath} \qquad \bullet P \qquad \frac{\sigma_1}{2\varepsilon_0}\hat{\imath} \qquad \frac{\sigma_3}{2\varepsilon_0}\hat{\imath}$$

$$\Rightarrow \vec{E}_{total} = \frac{1}{2\varepsilon_0}\left[|\sigma_1| + |\sigma_3| - |\sigma_2|\right]$$

$$= \frac{1}{2\varepsilon_0}\left[\sigma_1 + 2\sigma_1 - 3\sigma_1\right]$$

$$= \underline{0} \Rightarrow \boxed{E}$$

10). $U_i = 0$

$U_f = \dfrac{-kQq}{R}$, By the conservation of energy.

$\Delta U = -\Delta KE$

$\Rightarrow U_f - U_i = KE_i - KE_f$

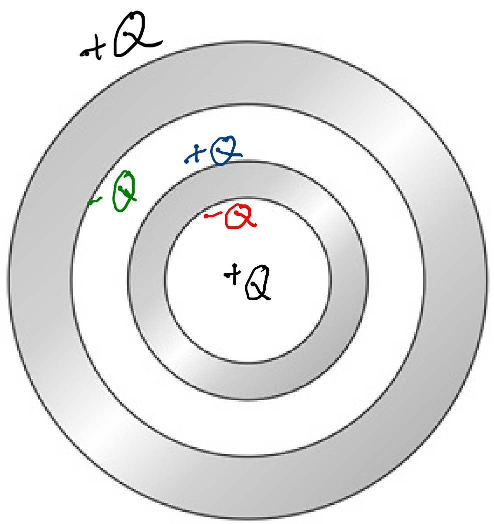
$\Rightarrow \dfrac{-kQq}{R} - 0 = 0 - KE_f$

$\Rightarrow \dfrac{kQq}{R} = KE_f = \dfrac{1}{2} m v_f^2$

$\Rightarrow v_f = \left(\dfrac{2kQq}{mR}\right)^{1/2} \Rightarrow \boxed{D}$

Free Response

1). a).



$\Rightarrow$

At $r = a$

$$\sigma_a = \frac{-Q}{4\pi a^2}$$

At $r = b$

$$\sigma_b = \frac{+Q}{4\pi b^2}$$

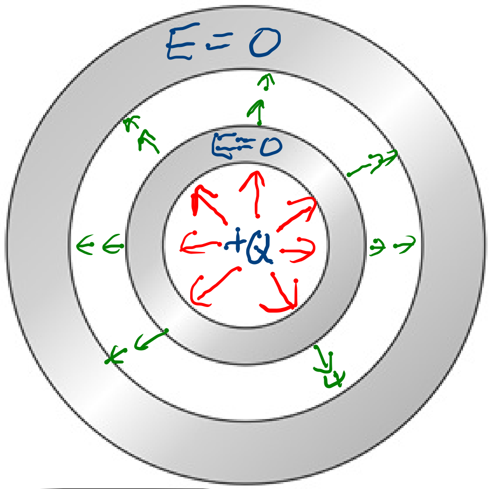
At $r = c$

$$\sigma_c = \frac{-Q}{4\pi c^2}$$

At $r = d$

$$\sigma_d = \frac{+Q}{4\pi d^2}$$

1). b

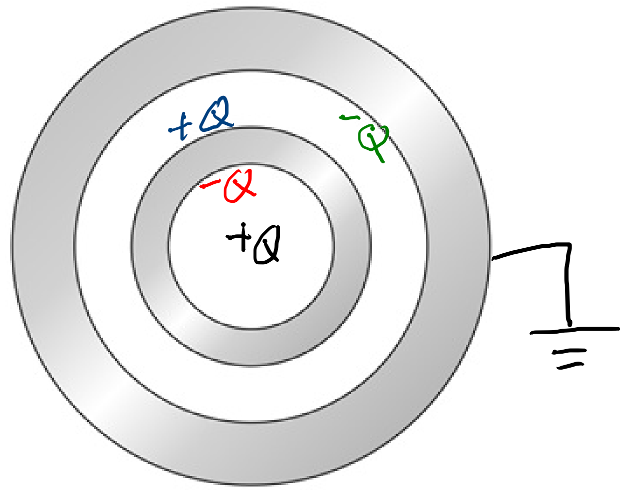


c). For $r > d$

$$\vec{E} = \frac{kQ}{r^2}\hat{r}$$

$$V = -\int_{\infty}^{r} \frac{kQ}{r^2}\hat{r} \cdot dr\,\hat{r} = -\left(-\frac{kQ}{r}\right|_{\infty}^{r} = \frac{kQ}{r}$$

1). d).



no charge on the outer surface of the outer shell because ΔV between $r=d$ & $r=\infty$ is zero.

So, For $r > d$

$$\vec{E} = 0$$

$\Rightarrow$ At $r=a$

$$\sigma_a = \frac{-Q}{4\pi a^2}$$

At $r=b$

$$\sigma_b = \frac{+Q}{4\pi b^2}$$

At $r=c$

$$\sigma_c = \frac{-Q}{4\pi c^2}$$

At $r=0$

$$\sigma_d = 0$$

For $b < r < c$

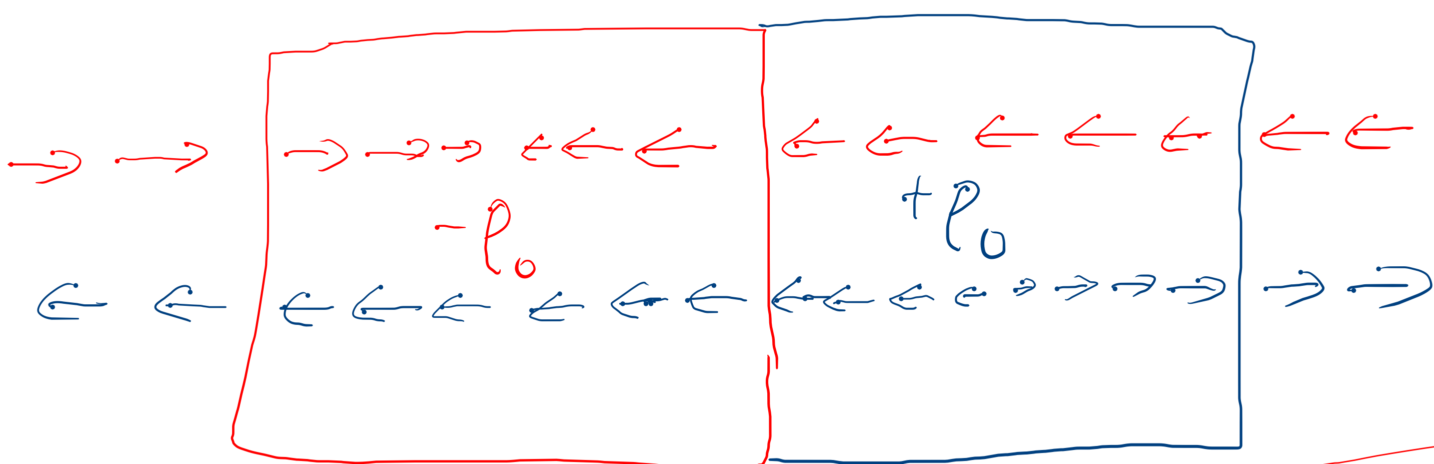
$$\vec{E} = \frac{kQ}{r^2}\hat{r}$$

$$\Rightarrow \Delta V = -\int_c^b \frac{kQ}{r^2}\,dr = \frac{kQ}{r}\Big|_c^b = \frac{kQ}{b} - \frac{kQ}{c}$$

a) + b)

2). By Gauss's Law, the magnitude of the electric field at the boundaries and outside of the boundaries of an infinite slab of charge (constant charge density) is constant.

Also, the magnitude of the electric field inside of an infinite slab of charge (constant charge density) increases linearly from zero at the center plane to the maximum value at the boundaries.



$\Rightarrow$ minimum at $x = -a$ & $x = +a$, maximum at $x = 0$

2).

c). using Gauss's law

At the boundary of an infinite slab of charge of constant charge density we get

$$2EA = \frac{\rho_0 A a}{\varepsilon_0} \Rightarrow E = \frac{\rho_0 a}{2\varepsilon_0}$$

so,

$$\boxed{\text{at } x=0 \quad E = \frac{-\rho_0 a}{\varepsilon_0}\hat{i}}$$

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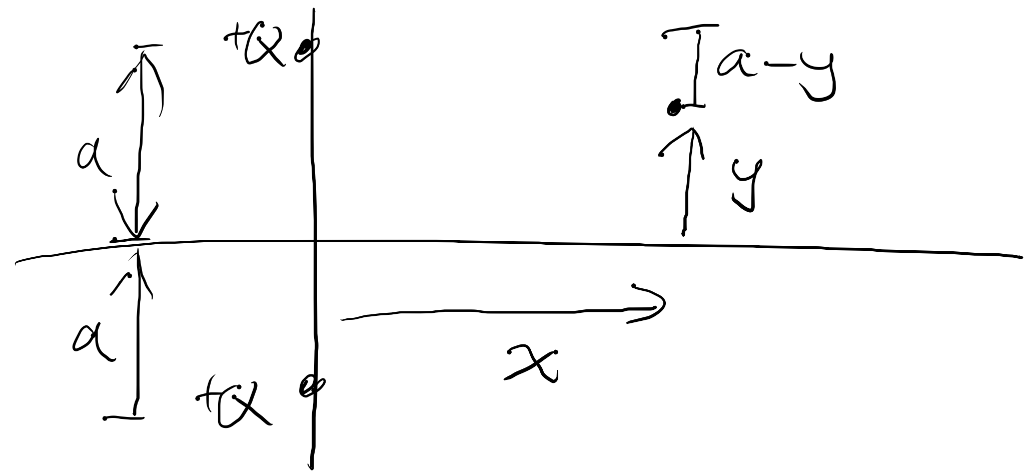
d).

$$\boxed{\text{at } x=\frac{a}{2} \quad E = \frac{-\rho_0 a}{2\varepsilon_0}\hat{i}}$$

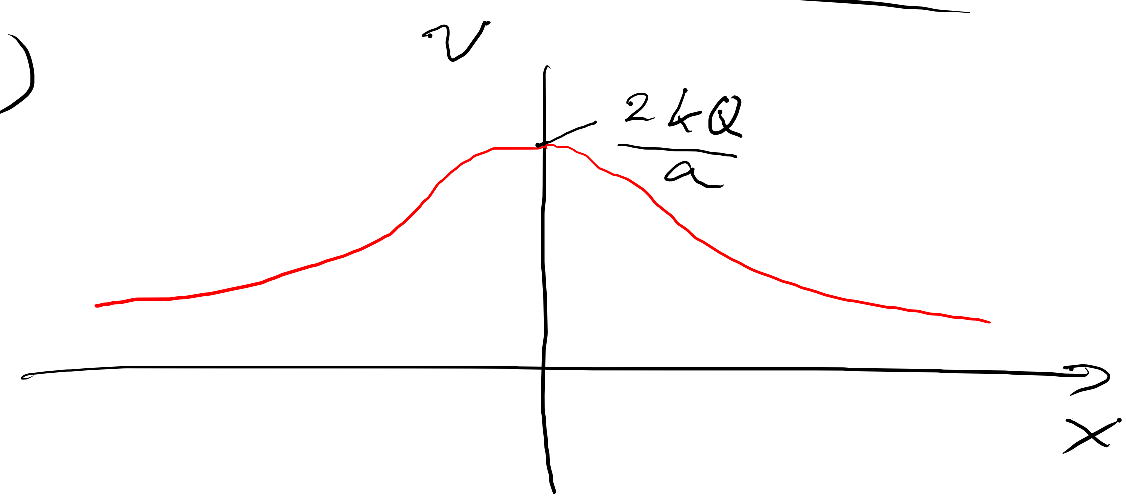
3). a).



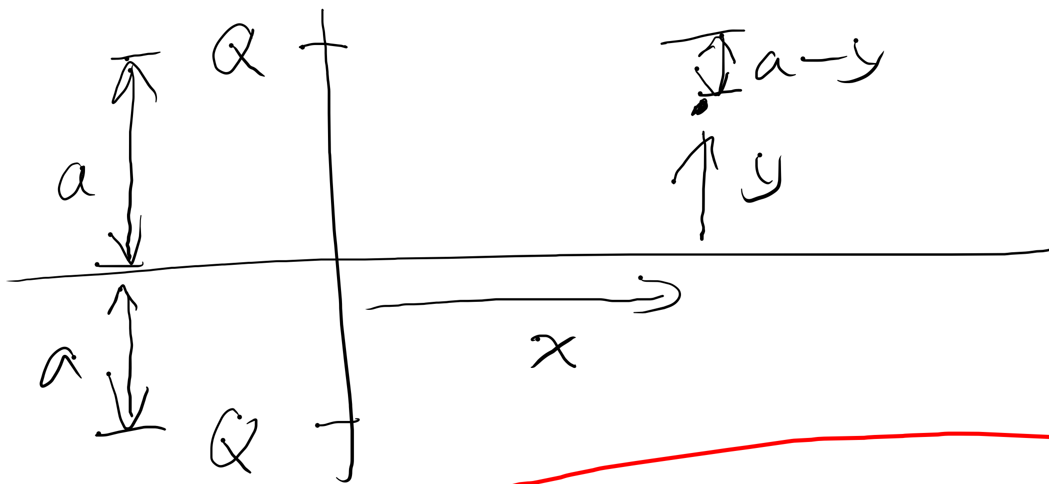
$$V = \frac{kQ}{\sqrt{x^2+(a-y)^2}} + \frac{kQ}{\sqrt{x^2+(a+y)^2}}$$

b). at $(x, y=0)$

$$V = \frac{2kQ}{\sqrt{x^2+a^2}}$$



3). c).



$$\vec{E} = \frac{kQ}{x^2+(a-y)^2}\left[\frac{x\,\hat{i} - (a-y)\,\hat{j}}{\sqrt{x^2+(a-y)^2}}\right] + \frac{kQ}{x^2+(a+y)^2}\left[\frac{x\,\hat{i} + (a+y)\,\hat{j}}{\sqrt{x^2+(a+y)^2}}\right]$$

$$\Rightarrow \vec{E} = kQx\left[\frac{1}{(x^2+(a-y)^2)^{3/2}} + \frac{1}{(x^2+(a+y)^2)^{3/2}}\right]\hat{i} + kQ\left[\frac{(a+y)}{(x^2+(a+y)^2)^{3/2}} - \frac{(a-y)}{(x^2+(a-y)^2)^{3/2}}\right]\hat{j}$$

3). d). By conservation of energy

$$\Delta U = -\Delta KE$$

$$\Rightarrow U_f - U_i = KE_i - KE_f$$

Here $U_i = \frac{kQ^2}{2a}$, $U_f = \frac{kQ^2}{5a}$, $KE_i = 0$

$$\Rightarrow \frac{kQ^2}{5a} - \frac{kQ^2}{2a} = 0 - KE_f$$

$$\Rightarrow KE_f = \frac{kQ^2}{a}\left[\frac{1}{2} - \frac{1}{5}\right] = \frac{kQ^2}{a}\left[\frac{3}{10}\right]$$