Phys 102 Final Exam SP23

Solution

Multiple-Choice

1). After the wire connected the two spheres

$$V_1 = V_2 \Rightarrow \frac{(Q_1)_f}{r_1} = \frac{(Q_2)_f}{2r_1}$$

$$\Rightarrow \boxed{2(Q_1)_f = (Q_2)_f}$$

By conserv. of charge

$$(Q_1)_i + (Q_2)_i = q + 2q = (Q_1)_f + (Q_2)_f$$

$$\Rightarrow 3q = 3(Q_1)_f \Rightarrow \boxed{(Q_1)_f = q}$$

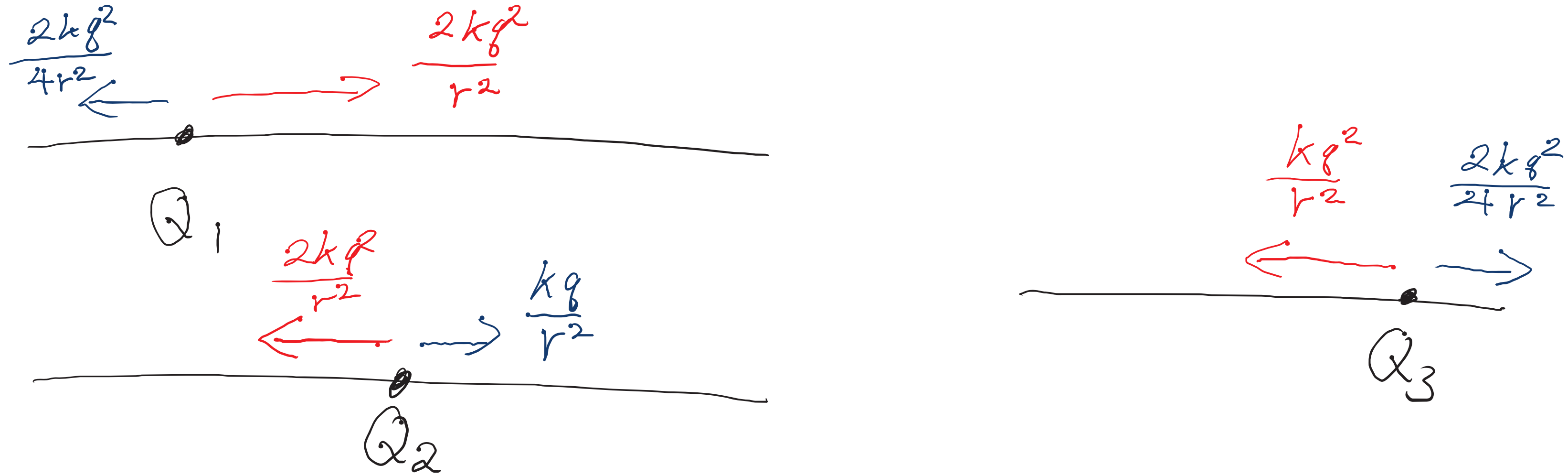
$$\Rightarrow \boxed{(Q_2)_f = 2q}$$

$$F = \frac{2kq^2}{R^2}$$

$$F_f = \frac{2kq^2}{R^2} \Rightarrow \boxed{F_f = F}$$

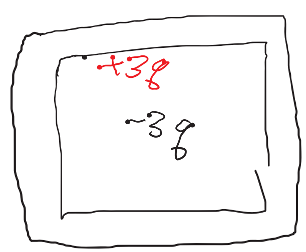
$$\boxed{D}$$

2).



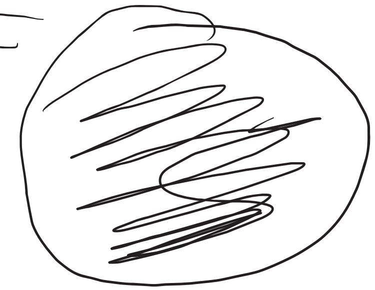
$\Rightarrow$ $\boxed{\text{right, left, left}}$ $\Rightarrow$ $\boxed{C}$

3). By Gauss's Law



$Q_{outer} = 0$ $\Rightarrow$ $\boxed{D}$

4). By superposition

$\vec{E}$  $=$ $\vec{E}$  $-$ $\vec{E}$ 

$$= \frac{k\rho\left(\frac{4}{3}\pi R^3\right)}{r^2}\hat{r} - \frac{k\rho\left(\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)}{r^2}\hat{r}$$

$$= \frac{4}{3}\frac{\rho}{r^2}\pi R^3\left[1-\frac{1}{2^3}\right]\hat{r}$$

$$= \frac{4}{3}\frac{\rho}{r^2}\pi R^3\left[\frac{7}{8}\right]\hat{r}$$

$$\Rightarrow \boxed{E_{\odot} = \frac{7}{8}E} \Rightarrow \boxed{A}$$

5). By Gauss's law, the electric field should be constant at $|x| \geq d$.

$$\Delta V = -\int \vec{E} \cdot d\vec{s} = -|\vec{E}|x$$

The only option that match is

$\boxed{D}$

---

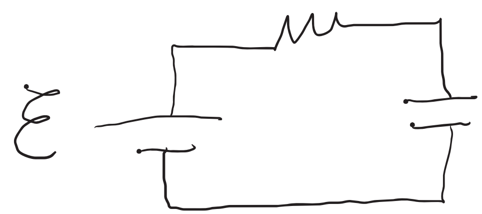
6). By Conservation of Energy.

$$\Delta KE = -\Delta U$$
$$= U_i - U_f$$
$$= QV_i - QV_f = Q[V_i - V_f]$$
$$= 7C[5V + 5V]$$
$$= \boxed{70J} \Rightarrow \boxed{C}$$

In between the plates, the electric potential increases linearly so, $V = (1\ \mathrm{V/cm})y - 10V$

7).

After $t=0$, the circuit behaves like



, Here $\mathcal{E} = \frac{d\Phi}{dt} = \frac{d}{dt}(B\ell^2)$

$\Rightarrow \mathcal{E} = (0.0005\,T/s)(0.2\,m)^2$

$\Rightarrow \underline{\mathcal{E} = 2\times 10^{-5}\,V}$.

The voltage across the capacitor is given by

$$\Delta V_c = \mathcal{E}\left[1 - e^{-t/RC}\right]$$

so, at $t = 15\,s$

$$\frac{1}{e} = 1 - e^{-\frac{15s}{RC}} \Rightarrow 1 - \frac{1}{e} = e^{-\frac{15\,s}{RC}}$$

$$\Rightarrow \frac{-15s}{RC} = \ln\left[1-\frac{1}{e}\right] \Rightarrow RC = \frac{-15\,s}{\ln\left[1-\frac{1}{e}\right]}$$

$$\Rightarrow \boxed{C = \frac{-15\,s}{R\ln\left[1-\frac{1}{e}\right]} = 3.3\times 10^{-4}\,F = 330\,\mu F}$$

$\Rightarrow \boxed{B}$

8). Here, Q is constance. $C_f = kC = 2C$

$$\Delta V_i = \frac{Q}{C}, \quad \Delta V_f = \frac{Q}{kC}$$

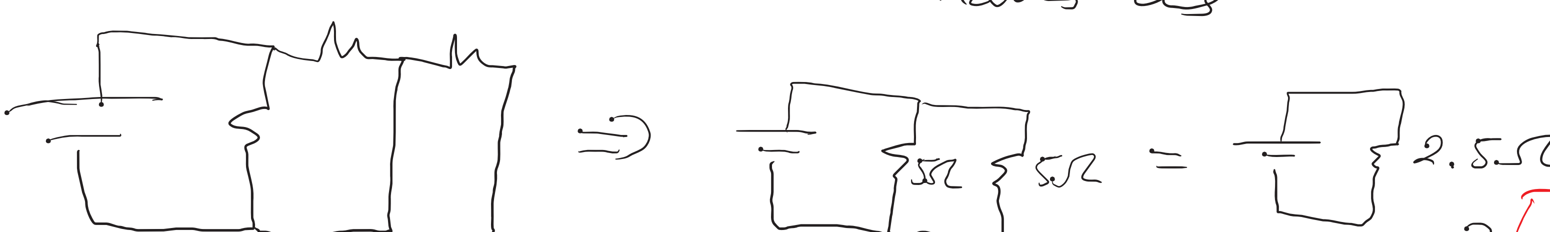
$$U_i = \frac{1}{2}\frac{Q^2}{C}, \quad U_f = \frac{1}{2}\frac{Q^2}{kC}$$

$$\boxed{E_i = \frac{Q}{Cd}, \quad E_f = \frac{Q}{kCd}}$$

$\Rightarrow \boxed{D}$

---

9). At $t=0$, when the switch closes, the circuit behaves as



$$\Rightarrow \boxed{I = \frac{16V}{2.5\Omega} = 4A}$$

$\Rightarrow \boxed{C}$

10). Using current rule

$$I_R = 1A + 2A = 3A$$

Using voltage rule

$$16V - 8V = 3A(R)$$

$$\Rightarrow \boxed{R = \frac{8V}{3A} = 2.67\,\Omega}$$

$$\Rightarrow \boxed{A}$$

---

11). By Newton's 3^{rd} Law

$$\boxed{|\vec{F}_1| = |\vec{F}_2|}$$

$$\Rightarrow \boxed{C}$$

12). As the current flows throught the spring, each loop of the spring generates a magnetic field.

Adjacent loops have magnetic polarity of the form

$$\boxed{N \quad S}\ \boxed{N \quad S}$$

or

$$\boxed{S \quad N}\ \boxed{S \quad N}$$

$\Rightarrow$ In either case, the cart would move to the left.

$\Rightarrow$ b

13). $\vec{\tau} = I\vec{A}\times\vec{B}$, Here $\vec{A}$ & $\vec{B}$ are perpendicular to each other for all three cases.

$$\Rightarrow \quad |\vec{\tau}|_{\square} = I a^2 B \; ; \quad |\vec{\tau}|_{\triangle} = I\left(\frac{1}{2}a^2\frac{\sqrt{3}}{2}\right)B = \frac{\sqrt{3}}{4} I a^2 B$$

$$|\vec{\tau}|_{\circ} = I\pi\left(\frac{a}{2}\right)^2 B = \frac{\pi}{4} I a^2 B$$

$$\Rightarrow \quad \boxed{|\vec{\tau}|_{\square} \text{ is the largest.}}$$

$$\Rightarrow \quad \boxed{A}$$

---

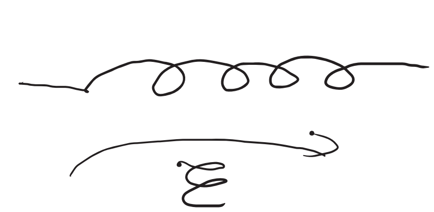
14). $\omega_i = \frac{1}{\sqrt{L_i C}}$, Here, $L_i = \frac{\mu_0 N}{\ell}\pi r^2$

$\omega_f = \frac{1}{\sqrt{L_f C}}$, Here $L_f = \frac{\mu_0 N}{2\ell}\pi r^2 = \frac{L_i}{2}$

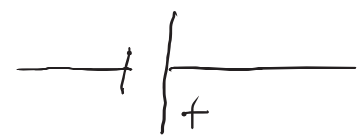
$$\Rightarrow \quad \boxed{\omega_f = \sqrt{2}\,\omega_i}$$

$$\Rightarrow \quad \boxed{C}$$

15). $\mathcal{E} = -\frac{d\Phi}{dt}$



Here, the inductor acts like a time varying battery with polarity



This means the current in the circuit is either

flowing to the right and decreasing

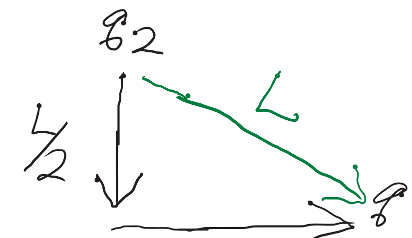
or

flowing to the left and increasing

$\Rightarrow \boxed{E}$

Free - Response Problems

1). a). $\vec{F}_1 = \vec{F}_{2 \text{ on } 1} + \vec{F}_{3 \text{ on } 1}$



$$\left(L^2 - \left(\frac{L}{2}\right)^2\right)^{\frac{1}{2}} = \frac{\sqrt{3}}{2} L$$

$$\Rightarrow \vec{F}_1 = \frac{k q_1 q_2}{L^2}\left[\frac{\frac{\sqrt{3}}{2} L \hat{i} - \frac{L}{2}\hat{j}}{L}\right] + \frac{k q_1 q_3}{L^2}\left[\frac{\frac{\sqrt{3}}{2} L \hat{i} + \frac{L}{2}\hat{j}}{L}\right]$$

$$\Rightarrow \vec{F}_1 = \frac{k q_1}{L^2}\frac{\sqrt{3}}{2}\left[q_2 + q_3\right]\hat{i} + \frac{k q_1}{L^2}\left(\frac{1}{2}\right)\left[q_3 - q_2\right]\hat{j}$$

Here $q_2 = q_3$

$$\Rightarrow \boxed{\vec{F}_1 = \frac{\sqrt{3}\, k q_1 q_2}{L^2}\hat{i} = \frac{\sqrt{3}\,(9\times10^{9})(-2\times10^{-6})(3\times10^{-6})}{(0.07)^2} N\,\hat{i} = -19.1\, N\,\hat{i}}$$

1). b). $\boxed{\vec{E}_1 = \dfrac{\vec{F}_1}{q_1} = \dfrac{-19.1N}{-2\times10^{-6}C}\,\hat{i} = 9.5\times10^{6}\,\dfrac{N}{C}\,\hat{i}}$

c). $$Work = \frac{k q_1 q_2}{L} + \frac{k q_1 q_3}{L} + \frac{k q_2 q_3}{L}$$

$$= \frac{k}{L}\left[q_1 q_2 + q_1 q_3 + q_2 q_3\right]$$

$$= \frac{9\times10^{9}}{0.07}\left[-2(3) + -2(3) + 3(3)\right]\times10^{-12}\,J$$

$$\Rightarrow \boxed{Work = -0.39\,J}$$

1). d). $\text{Work}_{ext} = \Delta U$

$= U_f - U_i$

$= \left(\frac{kq_1q_2}{\frac{L}{2}} + \frac{kq_1q_3}{L/2}\right) - \left(\frac{kq_1q_2}{L} + \frac{kq_1q_3}{L}\right)$

$= \frac{kq_1q_2}{L} + \frac{kq_1q_3}{L}$

$= \frac{kq_1}{L}\left[q_2 + q_3\right]$

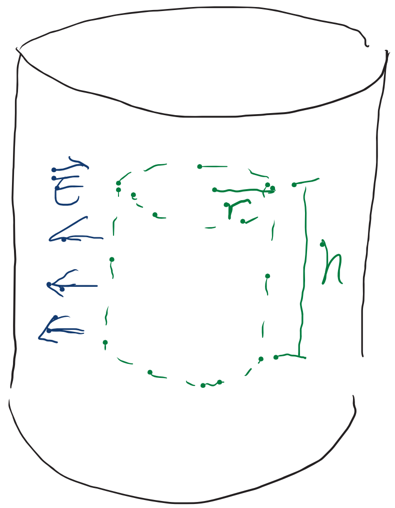
$= \frac{(9\times10^9)(-2)}{0.07}\left[3+3\right]\times10^{-12}\ \text{J}$

student can also use Work-KE theorem

$$\text{Work} = \int_{r=L}^{r=L/2} \vec{F}\cdot d\vec{r}$$

$\Rightarrow$ $\boxed{\text{Work}_{ext} = -1.54\ \text{J}}$

2). a).



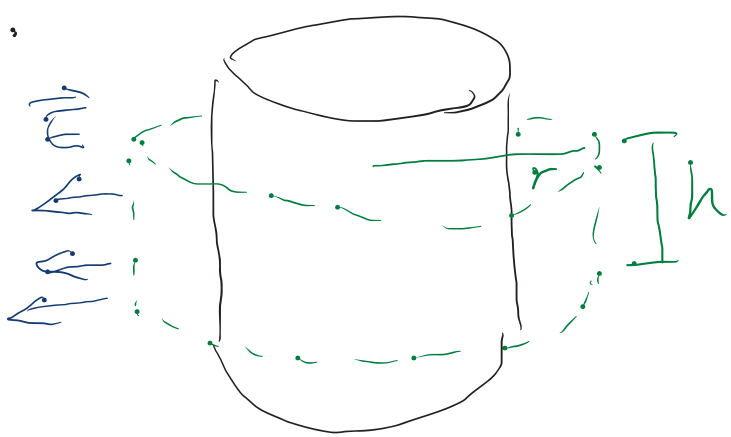
Using Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow |\vec{E}| \, 2\cancel{\pi} \cancel{r} \cancel{h} = \frac{\rho(\cancel{\pi} r^2 \cancel{h})}{\varepsilon_0}$$

$$\Rightarrow \boxed{|\vec{E}| = \frac{\rho r}{2\varepsilon_0}}$$

b).



Using Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow |\vec{E}| \, 2\cancel{\pi} r \cancel{h} = \frac{\rho(\cancel{\pi} a^2 \cancel{h})}{\varepsilon_0}$$

$$\Rightarrow \boxed{|\vec{E}| = \frac{\rho a^2}{2\varepsilon_0} \frac{1}{r}}$$

2). c).

$$\boxed{\begin{aligned}\vec{F}_Q &= Q\vec{E}(r=b)\\ &= \frac{Q\rho a^2}{2\varepsilon_0}\frac{1}{b}\,\hat{i}\end{aligned}}$$

---

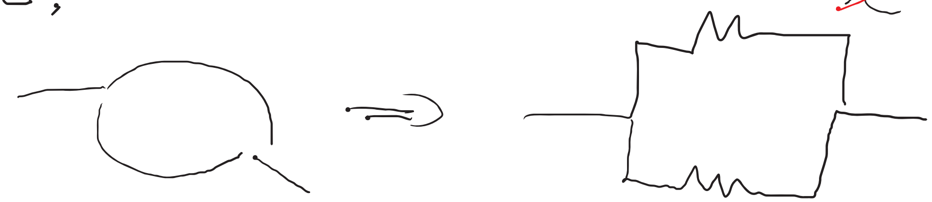
d). Using Work-KE theorem

$$KE = \int_{x=b}^{x=3b} Q\vec{E}\cdot dx\,\hat{i} = \int_{x=b}^{x=3b} \frac{Q\rho a^2}{2\varepsilon_0}\frac{dx}{x}$$

$$= \frac{Q\rho a^2}{2\varepsilon_0} Ln\left(\frac{3b}{b}\right)$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{Q\rho a^2}{2\varepsilon_0} Ln(3) \Rightarrow \boxed{v = \sqrt{\frac{Q\rho a^2 Ln(3)}{m\,\varepsilon_0}}}$$

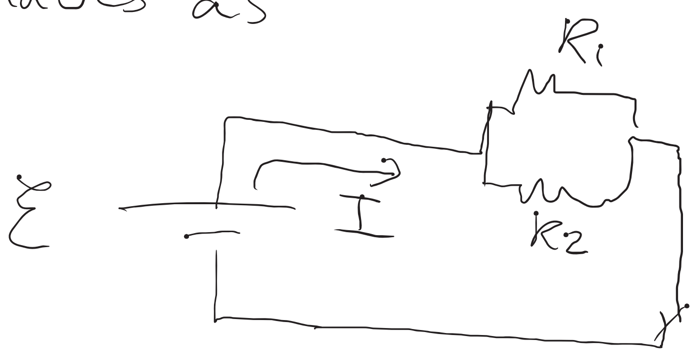
3). a). Here,



$$R_1 = \frac{1500\,\Omega}{\cancel{\pi}\,m} \cdot \frac{240}{360}\, 2\cancel{\pi}\, a = 20\,\Omega$$

$$R_2 = \frac{1500\,\Omega}{\cancel{\pi}\,m} \cdot \frac{120}{360}\, 2\cancel{\pi}\, a = 10\,\Omega$$

So, the circuit as $t \to \infty$ behaves as



As $t \to \infty$

$\Delta V_L \to 0$.

$$\Rightarrow \boxed{I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{\frac{R_1 R_2}{R_1 + R_2}} = \frac{20\,V}{\frac{(20)(10)}{30}\,\Omega} = 3\,A}$$

3). b). After $t=0$, the circuit is a DC-RL circuit

so,

$$I = I_{max}\left(1 - e^{-t\frac{R}{L}}\right)$$

Here,

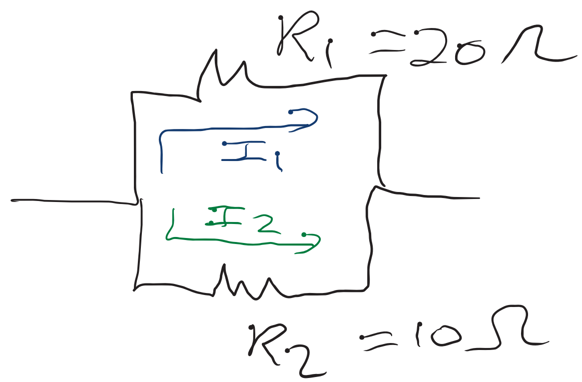
$$\Rightarrow \frac{I_{max}}{4} = I_{max}\left(1 - e^{-t_{1/4}R/L}\right)$$

$$\Rightarrow \frac{1}{4} = 1 - e^{-t_{1/4}\frac{R}{L}}$$

$$\Rightarrow 1 - \frac{1}{4} = e^{-t_{1/4}\frac{R}{L}} \Rightarrow \ln\left(\frac{3}{4}\right) = -\frac{R}{L} t_{1/4}$$

$$\Rightarrow \boxed{t_{1/4} = \frac{L}{R}\ln\left(\frac{4}{3}\right) = \frac{0.03\,H}{\frac{20}{3}\,\Omega}\ln\left(\frac{4}{3}\right) = 0.0013\,s}$$

3). c). For



Using the voltage rule

$$R_1 I_1 = R_2 I_2$$

$$\Rightarrow \quad I_1 = \frac{R_2}{R_1} I_2$$

The current rule tells us

$$I_1 + I_2 = I \quad \Longrightarrow \quad \frac{R_2}{R_1} I_2 + I_2 = I$$

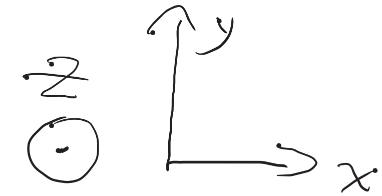
$$\Rightarrow \quad I_2 \left[\frac{R_2}{R_1} + 1 \right] = I \quad \Rightarrow \quad I_2 \left[\frac{R_1 + R_2}{R_1} \right] = I$$

$$\Rightarrow \quad \boxed{\frac{I_2}{I} = \frac{R_1}{R_1 + R_2} = \frac{20\Omega}{30\Omega} = \frac{2}{3}}$$

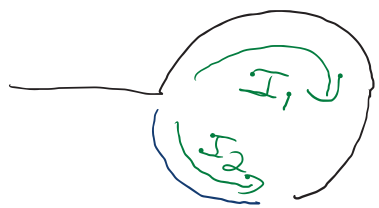
3). d). Biot-Savart law tells us

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\hat{r} \times \vec{L}}{r^2}$$

We use the Coord.



Here,



So,

$$\vec{B}_1 = \frac{\mu_0 I_1}{4\pi} \frac{2\pi \cancel{a} \left(\frac{240}{360}\right)}{a^{\cancel{2}}} \hat{k}$$

$$\vec{B}_2 = \frac{\mu_0 I_2}{4\pi} \frac{2\pi a \left(\frac{120}{360}\right)}{a^2} (-\hat{k})$$

$$\Rightarrow \quad \vec{B}_{total} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{2a}\left[\frac{2}{3} I_1 - \frac{1}{3} I_2\right] \hat{k}$$

Using the answer from part (c).

$$\boxed{\vec{B}_{total} = \frac{\mu_0}{2a}\left[\frac{2}{3}\left(\frac{I}{3}\right) - \frac{1}{3}\left(\frac{2I}{3}\right)\right]\hat{k} = 0}$$

3). e). Using Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}.$$

Here,

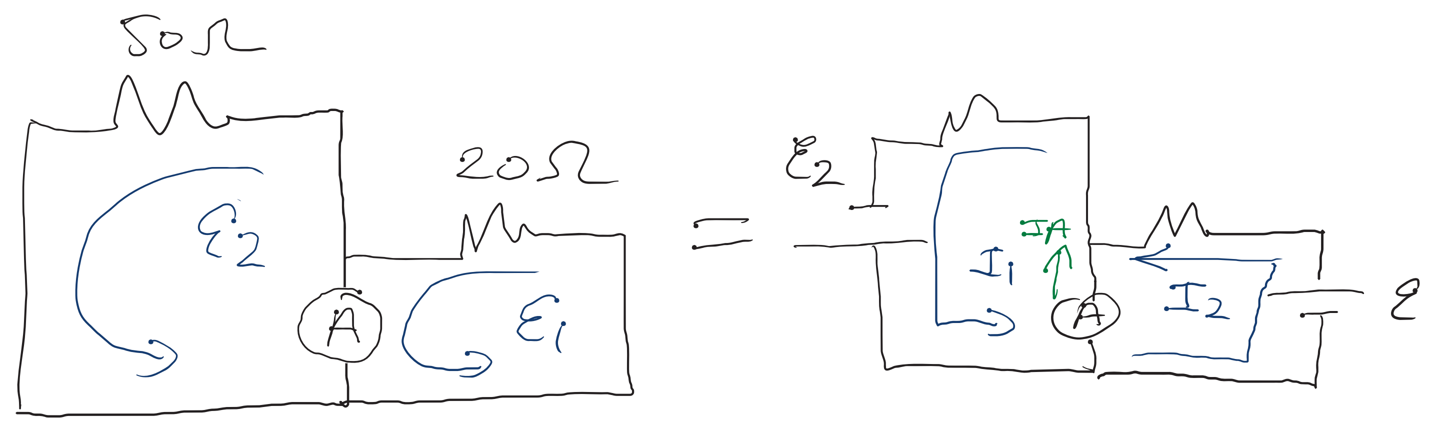
$$\oint \vec{B} \cdot d\vec{s} = \frac{\mu_0 I_{max}}{4} = \frac{\mu_0}{4} 3A$$

$$= \frac{4\pi \times 10^{-7}}{4} \frac{Tm}{A} (3A)$$

$$= 3\pi \times 10^{-7} \, Tm$$

$$\simeq 9.42 \times 10^{-7} \, Tm$$

4). a). As the magnetic field is changing the circuit behaves as



Here

$$\mathcal{E}_1 = \frac{-d\Phi_1}{dt} = \frac{-d}{dt}\left(-\beta t A_1\right) = \beta A_1$$

, where $A_1 = \frac{1}{3}\pi r^2$

$$\Rightarrow \quad \mathcal{E}_1 = \frac{\beta}{3}\pi r^2 \Rightarrow \boxed{I_1 = \frac{\beta \pi r^2}{3(20\Omega)} = \frac{\beta \pi r^2}{60\Omega}}$$

4). b). $\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -\frac{d}{dt}\left(-\beta t A_2\right) = \beta A_2$

, where $A_2 = \frac{2}{3}\pi r^2$

$\Rightarrow \quad \mathcal{E}_2 = \frac{2}{3}\beta\pi r^2 \Rightarrow \boxed{I_2 = \frac{2\beta\pi r^2}{3(50\Omega)} = \frac{\beta\pi r^2}{75\Omega}}$

---

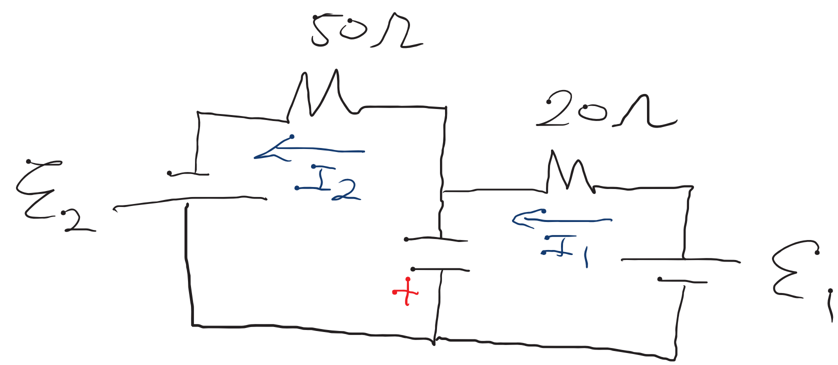
c). Here, by the current rule

$$I_2 = I_A + I_1$$

$$\Rightarrow I_A = I_2 - I_1 = \left(\frac{\beta\pi r^2}{60\Omega} - \frac{\beta\pi r^2}{75\Omega}\right)$$

$$\Rightarrow \boxed{I_A = \beta\pi r^2\left[\frac{1}{60\Omega} - \frac{1}{75\Omega}\right] = \frac{\beta\pi r^2}{300\Omega}}$$

4). d). Now, as the magnetic field is changing, the circuit behaves as



Using the voltage rule

$$\mathcal{E}_1 - I_1(20\Omega) - \Delta V_c = 0$$

$$\mathcal{E}_2 + \Delta V_c - I_2(50\Omega) = 0$$

When the capacitor is fully charged, $I_1 = I_2 = I$

$$\Rightarrow \Delta V_c = \mathcal{E}_1 - I(20\Omega) = I(50\Omega) - \mathcal{E}_2$$

$$\Rightarrow I(70\Omega) = \mathcal{E}_1 + \mathcal{E}_2 \Rightarrow \boxed{I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{70\Omega}}$$

4). d). $\Delta V_C = \mathcal{E}_1 - I(20\Omega)$

$$= \mathcal{E}_1 - \frac{\mathcal{E}_1 + \mathcal{E}_2}{70\Omega}(20\Omega)$$

$$= \mathcal{E}_1 - \frac{2}{7}\mathcal{E}_1 - \frac{2}{7}\mathcal{E}_2$$

$$= \frac{5}{7}\mathcal{E}_1 - \frac{2}{7}\mathcal{E}_2$$

$$= \frac{1}{7}\left[5\mathcal{E}_1 - 2\mathcal{E}_2\right]$$

$$\Rightarrow Q_{max} = C\Delta V_C = \frac{C}{7}\left[5\mathcal{E}_1 - 2\mathcal{E}_2\right] = \frac{C}{7}\left[5\frac{\beta\pi r^2}{3} - 2\left(\frac{2}{3}\right)\beta\pi r^2\right]$$

$$\Rightarrow \boxed{Q_{max} = \frac{\beta\pi r^2 C}{21}}$$