F22 Phys101 Midterm 2 Solution

Multiple-Choice

1).

$$\text{Power} = \frac{d(\text{Work})}{dt} = \frac{d}{dt}\int \vec{F}\cdot d\vec{x}\,,\quad \text{Here } \vec{F} \text{ is constant}$$

so,

$$\text{Power} = \frac{d}{dt}|\vec{F}|\cos\theta \int dx$$

$$= |\vec{F}|\cos\theta \frac{dx}{dt} = |\vec{F}|\, v\cos\theta$$

$$\Rightarrow \text{Power} = (80\,N)(0.5\,m/s)\cos(35^\circ) = 32.8\,W \Rightarrow \boxed{D}$$

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2).

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$$

$$\Rightarrow \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|} = \frac{3(5)+4(12)}{(5)(13)}$$

$$\Rightarrow \theta \approx \cos^{-1}[0.969] \Rightarrow \theta \approx 14.3^\circ \Rightarrow \boxed{B}$$

3). By conservation of mechanical energy

$$v_{B1} = v_{B2}$$

Also, the block on track 2 has a higher speed for more of the time than the block on track 1.

$\Rightarrow$ $\boxed{E}$

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4). By conservation of energy

$$(E_{total}) = U + KE = -3J + 1J = -2J$$

Since KE cannot be negative, it cannot reach where $U = 0$

$\Rightarrow$ $\boxed{E}$

5). By conservation of energy

$$\Delta U = -\Delta KE$$

$\Rightarrow$ $\boxed{E}$

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6). The explosion should not change the find position of the center-of-mass of the shell

$$R = \frac{\cancel{m}\left(\frac{R}{2}\right) + \cancel{m} x_{2f}}{2\cancel{m}} = \frac{R}{4} + \frac{x_{2f}}{2}$$

$$\Rightarrow \quad \frac{x_{2f}}{2} = \left(R - \frac{R}{4}\right) = \frac{3}{4}R$$

$$\Rightarrow \quad x_{2f} = \frac{3}{2}R \quad \Rightarrow \quad \boxed{B}$$

7). $F\Delta x$ is the same for ~~both~~ peas.

$$F\Delta x = \frac{1}{2} m v_1^2 \Rightarrow KE_1 = KE_2$$

$$F\Delta x = \frac{1}{2}(2m) v_2^2$$

$$\Rightarrow v_1^2 = 2v_2^2 \Rightarrow v_1 = \sqrt{2}\, v_2$$

$$\Rightarrow P_1 \neq P_2$$

$\boxed{C}$

8). This is a perfectly inelastic collision

$$mv_0 = (4m) v_f \Rightarrow v_f = \frac{v_0}{4}$$

$$\vec{J}_{car} = \Delta\vec{p} = m\frac{v_0}{4}\hat{i} - mv_0\hat{i} = mv_0\left[\frac{1}{4} - 1\right]\hat{i} = -\frac{3}{4} mv_0\hat{i}$$

$\Rightarrow \boxed{D}$

Using conservation of momentum.

9). along x-axis

$$m_0 v_0 \cos 60^\circ + m_0 v_0 \cos 60^\circ = 2m_0 (v_f)_x$$

along y-axis

$$m_0 v_0 \sin 60^\circ - m_0 v_0 \sin 60^\circ = 2m_0 (v_f)_y$$

$$\Rightarrow 0 = (v_f)_y$$

$$\Rightarrow (v_f) = (v_f)_x = \frac{2m_0 v_0 \cos 60^\circ}{2m_0} = v_0 \cos 60^\circ$$

$$\Rightarrow (v_f) = \frac{v_0}{2} \Rightarrow \boxed{B}$$

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10). The center-of-mass of the cannon + ball system is constant.

$$\Rightarrow 0 = \frac{m_c(-6m) + m_b(d)}{m_c + m_b}$$

$$\Rightarrow m_c(6m) = m_b(d) \Rightarrow d = \frac{m_c}{m_b}(6m) = \frac{400kg}{2kg}(6m)$$

$$\Rightarrow d = 1200\,m \Rightarrow \boxed{C}$$

Free-Response

1). a). $U_A = mgR = 0.2\,kg\,(9.8\,m/s^2)(0.5\,m)$

$$\Rightarrow \boxed{U_A = 0.98\,J}$$

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b). $KE_c = \frac{1}{2} m v_c^2 = \frac{1}{2}(0.2\,kg)(0.8\,m/s)^2$

$$\Rightarrow \boxed{KE_c = 0.064\,J}$$

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c). $\Delta U + \Delta KE + \Delta E_{int} = 0$

$$\Rightarrow \Delta E_{int} = -(\Delta U + \Delta KE)$$

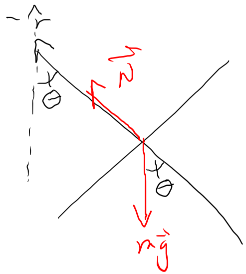
$$\boxed{\Delta E_{int} = 0.263\,J}$$

$$\Rightarrow \Delta E_{int} = (U_i - U_f) + (KE_i - KE_f)$$

$$\Rightarrow \Delta E_{int} = (U_A - U_c) + (KE_A - KE_c) \qquad \Uparrow$$

$$\Rightarrow \Delta E_{int} = \left[mgR - mg\left(\tfrac{2}{3}R\right)\right] + \left[0 - \tfrac{1}{2} m v_c^2\right] = \frac{mgR}{3} - \frac{1}{2} m v_c^2 = \frac{0.98\,J}{3} - 0.064\,J$$

1) d). FBD of cube at C

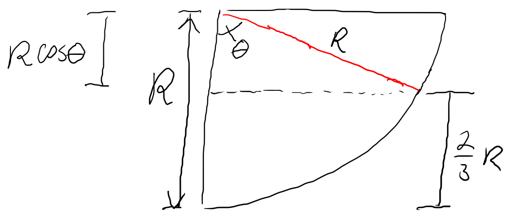


Along $-\hat{r}$ direction toward the center of the bowl

$$|\vec{N}| - mg\cos\theta = m a_r$$

$$\Rightarrow |\vec{N}| = \frac{m v_c^2}{R} + mg\cos\theta$$

To find $\cos\theta$



$$\Rightarrow R - R\cos\theta = \frac{2}{3}R \Rightarrow \cancel{R}[1-\cos\theta] = \frac{2}{3}\cancel{R}$$

$$\Rightarrow \boxed{\cos\theta = 1 - \frac{2}{3} = \frac{1}{3}}$$

$$\boxed{|\vec{N}| = m\left[\frac{v_c^2}{R} + \frac{g}{3}\right] = 0.2\,kg\left[\frac{(0.8\,m/s)^2}{0.5\,m} + \frac{9.8\,m/s^2}{3}\right] = 0.91\,N}$$

2). a). Using conservation of energy

$$\Delta U_{spring} = -\Delta KE$$

$$\left(0 - \frac{1}{2}k\Delta x^2\right) = -\left(\frac{1}{2}m_1 v^2 - 0\right)$$

$$\Rightarrow \quad k\Delta x^2 = m_1 v_1^2 \quad \Rightarrow \quad v_1 = \Delta x \sqrt{\frac{k}{m_1}}$$

$$\Rightarrow \quad \boxed{v_1 = 0.2\,m \sqrt{\frac{200\,N/m}{0.5\,kg}} = 4\,m/s}$$

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b).

$$\vec{v}_{cm} = \frac{m_1 v_1 \hat{i} - m_2 v_2 \hat{i}}{m_1 + m_2} = \frac{0.5\,kg\,(4\,m/s)\,\hat{i} - 2\,kg\,(0.5\,m/s)\,\hat{i}}{2.5\,kg}$$

$$\Rightarrow \quad \vec{v}_{cm} = 0.4\,m/s\,\hat{i} \quad \Rightarrow \quad \boxed{|\vec{v}_{cm}| = 0.4\,m/s}$$

2). c) When the block reaches highest point on the inclined plane the relative speed between the block and the inclined plane is <u>zero</u>.

So, the block & the inclined plane will have the same velocity.

Using conservation of momentum along the x-axis

$$m_1 v_1 \hat{i} - m_2 v_2 \hat{i} = (m_1 + m_2) \vec{v_f}$$

$$\Rightarrow \quad \vec{v_f} = \frac{m_1 v_1 - m_2 v_2}{(m_1 + m_2)} \hat{i}$$

$$\Rightarrow \quad |\vec{v_f}| = \frac{0.5\,kg\,(4\,m/s) - 2\,kg\,(0.5\,m/s)}{2.5\,kg}$$

$$\Rightarrow \quad \boxed{|\vec{v_f}| = 0.4\,m/s}$$

2). d). Using conservation of energy.

$$\Delta U_g + \Delta U_{spring} + \Delta KE = 0$$

$$\Rightarrow (m_1 g h_{max} - 0) + \left(0 - \frac{1}{2} k \Delta x^2\right) + \left(\frac{1}{2}(m_1+m_2) v_f^2 - \frac{1}{2} m_2 v_2^2\right) = 0$$

$$\Rightarrow m_1 g h_{max} = \frac{1}{2} k \Delta x^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2}(m_1+m_2) v_f^2$$

$$\Rightarrow h_{max} = \frac{1}{2 m_1 g}\left[k \Delta x^2 + m_2 v_2^2 - (m_1+m_2) v_f^2\right]$$

$$\Rightarrow h_{max} = \frac{1}{2(0.5kg)(9.8 m/s^2)}\left[200 \frac{N}{m}(0.2m)^2 + 2kg(0.5 m/s)^2 - (2.5kg)(0.4 m/s)^2\right]$$

$$\Rightarrow \boxed{h_{max} = 0.83 m}$$

2). e). $Work = \Delta KE$

$\Rightarrow Work = \frac{1}{2} m_1 v_f^2 - \frac{1}{2} m_1 v_1^2$

$\Rightarrow Work = \frac{m_1}{2} [v_f^2 - v_1^2]$

$\Rightarrow Work = \frac{0.5 kg}{2} \left[(0.4 m/s)^2 - (4 m/s)^2 \right]$

$\Rightarrow \boxed{Work = -3.96 \ J}$

3). a).

$$P_1 = m_1 v_1 \Rightarrow v_1 = \frac{P_1}{m_1} = \frac{2\ kg\ m/s}{0.5\ kg}$$

$$\Rightarrow \boxed{v_1 = 4\ m/s}$$

b). Using conservation of energy & Recall $KE = \frac{p^2}{2m}$

$$\frac{1}{2} m_1 v_0^2 = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2}$$

$$\Rightarrow \frac{P_2^2}{2m_2} = \frac{m_1 v_0^2}{2} - \frac{P_1^2}{2m_1} \Rightarrow P_2^2 = m_1 m_2 v_0^2 - \frac{m_2}{m_1} P_1^2$$

$$\Rightarrow P_2 = \sqrt{(0.5\ kg)(1.4)(0.5\ kg)(10\ m/s)^2 - 1.4(2\ kg\ m/s)^2}$$

$$\Rightarrow \boxed{P_2 = 5.42\ kg\ m/s}$$

3). c).

$$\vec{I} = \Delta\vec{p} = \vec{p}_1 - \vec{p}_0$$

and by conserv. of momentum

$$\vec{p}_0 = \vec{p}_1 + \vec{p}_2 \Rightarrow \vec{p}_1 = \vec{p}_0 - \vec{p}_2$$

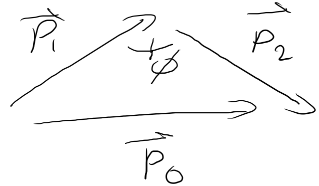
$$\vec{I} = \vec{p}_0 - \vec{p}_2 - \vec{p}_0 = -\vec{p}_2$$

$$\Rightarrow \boxed{|\vec{I}| = |\vec{p}_2| = 5.42\ kg\,m/s}$$

3). d) According to the conservation of momentum

$$\vec{P}_0 = \vec{P}_1 + \vec{P}_2$$

$\Rightarrow$



Using the Law of Cosine

$$P_0^2 = P_1^2 + P_2^2 - 2P_1P_2\cos\phi$$

$$\Rightarrow \quad 2P_1P_2\cos\phi = P_1^2 + P_2^2 - P_0^2$$

$$\Rightarrow \quad \cos\phi = \frac{P_1^2 + P_2^2 - P_0^2}{2P_1P_2} = \frac{(2\,\mathrm{kg\,m/s})^2 + (5.42\,\mathrm{kg\,m/s})^2 - (0.5\,\mathrm{kg}\cdot 10\,\mathrm{m/s})^2}{2\,(2\,\mathrm{kg\,m/s})\,(5.42\,\mathrm{kg\,m/s})}$$

$$\Rightarrow \quad \cos\phi = 0.386$$

$$\Rightarrow \quad \phi \simeq 67.27^\circ \Rightarrow \Theta = 180^\circ - \phi$$

$$\Rightarrow \quad \boxed{\Theta = 112.73^\circ}$$