Midterm 1 Fall 2022 Solution

Multiple-Choice

1).

$$a_1 = \frac{F}{m}, \qquad a_2 = \frac{F}{2m}$$

$$d = \frac{1}{2}\frac{F}{m}t^2 \Rightarrow t^2 = \frac{2dm}{F}$$

$$\Rightarrow d_2 = \frac{1}{2}\left(\frac{F}{2m}\right)\left(\frac{2dm}{F}\right) = \frac{d}{2} \Rightarrow \boxed{B}$$

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2). Let $t=0$ be when the 1^{st} object was dropped and d be the separation between the 2 objects.

For $t \geq 1\,s$

$$d = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1s)^2$$

$$\Rightarrow d = \frac{1}{2}gt^2 - \frac{1}{2}g\left(t^2+(1s)^2-2t(1s)\right)$$

$$\Rightarrow d = gt(1s) - \frac{g}{2}(1s)^2 = g(1s)\left[t - \frac{1}{2}s\right]$$

$\Rightarrow$ d increases linearly with time $\Rightarrow \boxed{A}$

3).

$$\vec{a}_{1 \text{ to } 2} = \vec{a}_{1 \text{ to road}} + \vec{a}_{\text{road to } 2}$$

$$= \longrightarrow + \downarrow$$

$$= \searrow \Rightarrow \boxed{C}$$

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4). Here, $-y_i = (v_y)_i t_f - \frac{g}{2} t_f^2$

$$\Rightarrow \frac{g}{2} t_f^2 - (v_y)_i t_f - y_i = 0$$

$$\Rightarrow t_f = \frac{(v_y)_i + \sqrt{(v_y)_i^2 + 2 g y_i}}{g}$$

And

$$(v_y)_f = (v_y)_i - g t_f = (v_y)_i - (v_y)_i - \sqrt{(v_y)_i^2 + 2 g y_i}$$

$$\Rightarrow (v_y)_f = -\sqrt{(v_y)_i^2 + 2 g y_i}$$

Case A has the largest $(v_y)_i$, which means it also has the larges $|(v_y)_f|$

$$\Rightarrow \boxed{A}$$

5). FBD of mass

$\uparrow \vec{T}$ $\uparrow \vec{a}$

$\downarrow m\vec{g}$

$\Rightarrow |\vec{T}| - mg = ma$

$\Rightarrow |\vec{T}| = ma + mg = m[a+g]$

$\Rightarrow |\vec{T}| = 5 kg [12 m/s^2] = 60N$

$\Rightarrow \boxed{C}$

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6). Here, the stone traveled a longer horizontal distance than expected if the train were stationary. This means the stone experiences a fictitious force to the right.

So, the train car must be accelerating to the left

$\Rightarrow \boxed{E}$

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7). I. At B, $\vec{a} = -g\hat{j}$

II. At A, $|\vec{v}_A| = |\vec{v}_i|$, at C, $|\vec{v}_C| = |\vec{v}_i| \Rightarrow |\vec{v}_A| = |\vec{v}_C|$ ✓

III. V_x is constant $= |\vec{v}_i| \cos\theta$

$\Rightarrow \boxed{B}$

8). I. At $t = t_A$, $v > 0$ because $\Delta v = \int_0^t a_x dt$

II. At $t = t_B$, $v > 0$

III. Since $v > 0$ for $0 < t < t_c$

and $\Delta x = \int_0^t v_x dt$

$\Delta x > 0$

$\Rightarrow \boxed{E}$

9). For uniform circular motion,

$\vec{a}$ points toward the center of the circle

$\Rightarrow \boxed{B}$

10). I. FBD of crate

$\uparrow \vec{N}$ $\rightarrow \vec{F}_{\text{truck on crate}}$

$\downarrow m\vec{g}$

II. The road pushes the truck in the $+x$ direction, and the truck pushes the road in the $-x$ direction

III. $\vec{F}_{net} = m\vec{a}$. If $\vec{a} \neq 0$, then $\vec{F}_{net} \neq 0$.

$\Rightarrow \boxed{E}$

Free Response

1). a). $|\vec{a}| = \sqrt{g^2 + a_x^2} = \sqrt{(9.8\,m/s^2)^2 + (2\,m/s^2)^2} = \boxed{10\ m/s^2}$

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b). using $y_f = y_i - \frac{g}{2}t^2$, with $y_i = 300\,m$ & $y_f = 0\,m$

$\Rightarrow \quad 0 = 300\,m - \frac{g}{2}t_f^2 \rightarrow t_f = \sqrt{\frac{2(300\,m)}{g}}$

$\Rightarrow \quad \boxed{t_f = 7.82\,s}$

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c). using $D = \frac{a_x}{2}t_f^2 = \frac{2\,m/s^2}{2}(7.82\,s)^2$

$\Rightarrow \quad \boxed{D = 61.22\,m}$

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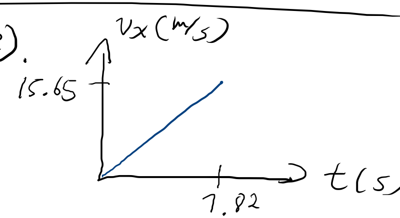
d). $(v_x)_f = a_x t_f$, $(v_y)_f = -g t_f$

$= 15.65\,m/s$ $\qquad = -76.68\,m/s$

$\Rightarrow \quad \boxed{\vec{v}_f = 15.65\,m/s\,\hat{i} - 76.68\,m/s\,\hat{j}}$

e).



2). a). At $t = 1\,s$, $F_x = 3.5N$

$$\Rightarrow \boxed{a_x = \frac{F_x}{m} = \frac{3.5N}{5kg} = 0.7\,m/s^2}$$

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b).

$$\Delta v_x = \int_0^{t=1.75s} a_x\,dt.$$ Here

$$\Delta v_x = \frac{1s\left[\frac{1.5N + 3.5N}{5kg}\right]}{2} + 0.75\left[\frac{3.5N}{5kg}\right]$$

$$\Rightarrow \boxed{v_x(t=1.75s) = 1.025\,m/s}$$

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c). After $t=0$, the object reaches its maximum displacement along the x-axis when $v_x = 0$.

This occurs when

$$1.025\,m/s + \frac{1}{2}(0.5s)\left(\frac{3.5N}{5kg}\right) = \frac{3.5N}{5kg}\left[\frac{(t_m - 2.25s) + (t_m - 2.75s)}{2}\right]$$

$$\Rightarrow 1.2\,m/s = 0.35\,m/s^2\,[2t_m - 5s] \Rightarrow \boxed{t_m = 4.21\,s}$$

Alternate solution to FR 2(b) & 2(c)

2(b) We note each square represents

$$\Delta v = \frac{(0.5N)(0.25s)}{5kg} = 0.025 \, m/s$$

From $t=0$ to $t=1.75s$, there are approx

$4(7) + 13 = 41$ squares

So,

$$\boxed{\Delta v \Big|_{t=0}^{t=1.75s} \approx (41)(0.025 \, m/s) = 1.025 \, m/s}$$

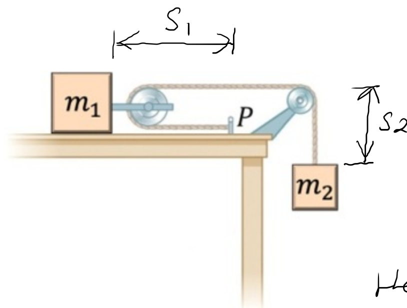
2(c) From $t=0$ to $t=2.25s$, there are approx 48 squares

So, $\Delta v \Big|_{t=0}^{t=2.25s} \approx (48)(0.025 \, m/s) = 1.2 \, m/s$

From $t=2.25s$ to $t=4.25s$, there are approx. $(7)(6) + 7 = 49$ squares

So, $\boxed{t_{max} \approx 4.2s}$

3). a).



Here

$$2S_1 + S_2 = \text{Constant}$$

$$\Rightarrow \frac{d^2}{dt^2}[2S_1+S_2] = \frac{d}{dt^2}[\text{constant}]$$

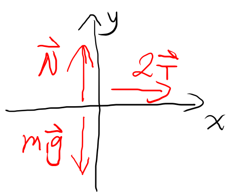
$$\Rightarrow 2\frac{d^2S_1}{dt^2} + \frac{d^2S_2}{dt^2} = 0$$

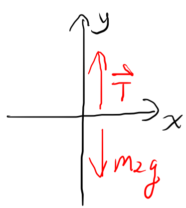
Here $\frac{d^2S_1}{dt^2} = a_1$ & $\frac{d^2S_2}{dt^2} = a_2$

$$\Rightarrow 2a_1 + a_2 = 0 \Rightarrow 2a_1 = -a_2$$

$$\Rightarrow \boxed{\frac{|\vec{a}_1|}{|\vec{a}_2|} = \frac{1}{2}}$$

b). FBD m_1 FBD m_2





c). From FBD of m_1

$$\Sigma F_x = 2|\vec{T}| = m_1 a_1$$

From FBD of m_2

$$\Sigma F_y = |\vec{T}| - m_2 g = -m_2 a_2$$

$$|\vec{T}| = \frac{m_1 a_1}{2} = \frac{m_1 a_2}{4} \Rightarrow \frac{m_1 a_2}{4} - m_2 g = -m_2 a_2$$

$$\Rightarrow a_2\left[\frac{m_1}{4} + m_2\right] = m_2 g \Rightarrow \boxed{a_2 = \frac{m_2 g}{\left[\frac{m_1}{4} + m_2\right]}}$$

$$\Rightarrow \boxed{a_2 = \frac{g}{\left[\frac{m_1}{4m_2} + 1\right]}}$$

d).

$$\boxed{|\vec{T}| = \frac{m_1 a_2}{4} = \frac{m_1}{4}\left[\frac{m_2 g}{\left[\frac{m_1}{4}+m_2\right]}\right]}$$

$$\Rightarrow \boxed{|\vec{T}| = \frac{m_1 m_2 g}{m_1 + 4m_2} = \frac{m_1 g}{\left[\frac{m_1}{m_2}+4\right]}}$$