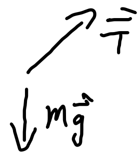
F22 Phys101 Final Exam Solution

Multiple-Choice

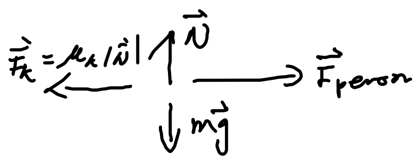
1). $\boxed{C}$ The acceleration due to the gravity of the Earth is independent to the mass of the object.

2). $\boxed{B}$ FBD of the person



I & II

3). $\boxed{C}$ FBD of the box



$\Rightarrow |\vec{F}_{person}| = \mu_k |\vec{N}| = \mu_k mg$

since $\mu_k < 1$

$mg > |\vec{F}_{person}|$

4). $\boxed{E}$

$$\vec{v}_{puck} = v_0\hat{i} + v_k\hat{j}$$

$$\Rightarrow |\vec{v}_{puck}| = \sqrt{v_0^2 + v_k^2} > v_0 \ \& \ > v_k$$

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5). $\boxed{D}$

$$\vec{v}_{rocket} = v_0\hat{i} + at\hat{j}$$

$$\vec{r}_{rocket} = v_0 t\hat{i} + \frac{at^2}{2}\hat{j}$$

$$\Rightarrow \quad x = v_0 t \Rightarrow t = \frac{x}{v_0}$$

$$y = \frac{at^2}{2} \longrightarrow y = \frac{a}{2}\frac{x^2}{v_0^2}$$

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6). $\boxed{C}$

Work $= \Delta KE$, since the satellite moves with a constant speed

$$\Delta KE = 0$$

7). $\boxed{C}$

FBD of block

Case (1)

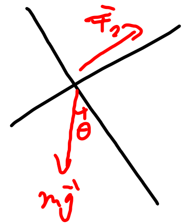
$\uparrow \vec{T}_1$ $\quad \vec{a} = 0$

$\downarrow m\vec{g}$

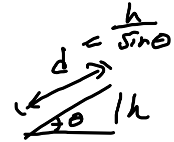
$\Rightarrow |\vec{T}_1| = mg$

$Work_T = |\vec{T}_1| h = mgh$

Case (2)



$\vec{a} = 0$



$\Rightarrow |\vec{T}_2| = mg\sin\theta$

$Work_T = |\vec{T}_2| d = mg\sin\theta \frac{h}{\sin\theta}$

$= mgh$

8). $\boxed{B}$

For the ball

$\Delta U_g = -\Delta KE$

$\Rightarrow mg\Delta y = \frac{1}{2} m v^2$

9). $\boxed{B}$

$\vec{J} = \vec{F}_{AVG} \Delta t = \Delta \vec{p}$

$\Rightarrow \vec{F}_{AVG} = \frac{\Delta \vec{p}}{\Delta t}$

As Δt increases, $\vec{F}_{AVG}$ decreases

10). $\boxed{D}$

An explosion is a perfect inelastic collision in reverse.

This meas momentum is conserved, but kinetic energy is not conserved.

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11). $\boxed{D}$

$$|\vec{F}| = \frac{GMm}{r^2} \Rightarrow a = \frac{GM}{r^2}$$

$$a_{surface} = \frac{GM}{R^2}, \quad a_{top} = \frac{GM}{(2R)^2}$$

$$\boxed{a_{top} = \frac{a_{surface}}{4}}$$

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12). $\boxed{D}$

No slipping means $V_A = V_B \Rightarrow \cancel{r}\omega_A = 3\cancel{r}\omega_B$

$\Rightarrow \omega_A = 3\omega_B$

$a_A = r\omega_A^2 = r(3\omega_B)^2 = 9r\omega_B^2 \rightarrow \boxed{a_A = 3a_B}$

$a_B = 3r\omega_B^2$

13). $\boxed{E}$

For constant speed.

$$\text{Power} = F v$$

$$\Rightarrow \text{Power} = \beta v^3 \Rightarrow \boxed{\frac{\text{Power}_{60}}{\text{Power}_{30}} = \left(\frac{60}{30}\right)^3 = 2^3 = \underline{8}}.$$

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14). $\boxed{E}$

Here

$$m_{small} < M_{Large}$$

$$F_{small} = \frac{G M_{Large} m_{small}}{r^2} = F_{Large} = \frac{G m_{small} M_{Large}}{r^2}$$

$$\Rightarrow a_{small} = \frac{G M_{Large}}{r^2} \; ; \; a_{Large} = \frac{G M_{small}}{r^2}$$

$$\Rightarrow a_{small} > a_{Large}.$$

$$\Rightarrow V_{small} > V_{Large}.$$

$$\Rightarrow \Delta x_{small} > \Delta x_{Large}$$

$$\boxed{\Delta P_{small} = \int_0^{\Delta t} F_{small}\, dt = \int_0^{\Delta t} F_{Large}\, dt = \Delta P_{Large}}$$

$$KE_{small} = \int_0^{\Delta x_{small}} F_{small}\, dx > \int_0^{\Delta x_{Large}} F_{Large}\, dx = KE_{Large}$$

15). $\boxed{A}$

Let $x_1 \equiv$ Distance from cm of 1 to the cm of the bat
$x_2 \equiv$ Distance from cm of 2 to the cm of the bat

Using the cm of the bat as the origin

$$\frac{m_2 x_2 - m_1 x_1}{m_2 + m_1} = 0$$

$$\Rightarrow \quad m_2 x_2 = m_1 x_1$$

$$\Rightarrow \quad \underline{m_1 = m_2 \left(\frac{x_2}{x_1}\right)}$$

Assuming the bat is made with a single material,

$$x_2 > x_1$$

$$\Rightarrow \underline{m_1 > m_2}$$

16). $\boxed{E}$

system = $M + m$

$$\vec{F}_{net} = M_{total}\,\vec{a}_{cm}$$

$$\vec{a}_{cm} = \frac{\vec{F}_{net}}{M_{total}}$$

Here $\vec{F}_{net} = m\vec{g}$

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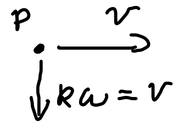
17). $\boxed{D}$

Using the parallel-axis theorem

$$I = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

18). $\boxed{B}$



$$|\vec{v}_p| = \sqrt{v^2 + v^2} = \sqrt{2}\, v$$

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19). $\boxed{D}$

The pivot is to the left of the CM of the picture frame.

The picture frame is stable when the pivot is directly above the CM of the picture frame.

So, The CM of the picture frame will move down & leftward.

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20). $\boxed{A}$

$$x = A\cos(\omega t) = A\cos(2\pi f t)$$

$$v = -2\pi f A \sin(2\pi f t) \qquad \omega = 2\pi f$$

$$v_{max} = 2\pi f A = 2\pi (10\,Hz)(0.03\,m) = \underline{1.88\,m/s}$$

Free-Response

1). a).

$$D = \Delta x_1 + \Delta x_2 = \frac{a_1}{2}t^2 + (0.01\ m/s^3)t^3$$

$$\Rightarrow \frac{a_1}{2}t^2 = D - (0.01\ m/s^3)t^3$$

$$\Rightarrow a_1 = \frac{2}{t^2}\left(D - (0.01\ m/s^3)t^3\right)$$

$$\Rightarrow a_1 = \frac{2}{(35s)^2}\left(1000\ m - (0.01\ m/s^3)(35s)^3\right)$$

$$\Rightarrow \boxed{a_1 = 0.9327\ m/s^2}$$

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b).

$$V_2 = \frac{d(x_2)}{dt} = 3(0.01\ m/s^3)t^2$$

$$\Rightarrow \boxed{V_2 = 3(0.01\ m/s^3)(35s)^2 = 36.75\ m/s}$$

1). c). $v_1 = a_1(35s) = (0.9327\ m/s^2)(35s)$

$\Rightarrow v_1 = 32.643\ m/s$

$$\vec{V}_{cm} = \frac{m_1 v_1 \hat{i} - m_2 v_2 \hat{i}}{m_1 + m_2} = \frac{(0.1kg)(32.643\,m/s) - (0.2kg)(36.75\,m/s)}{(0.1kg + 0.2kg)}\hat{i}$$

$\Rightarrow \vec{V}_{cm} = -13.619\ m/s\ \hat{i}$

$\Rightarrow \boxed{|\vec{V}_{cm}| = 13.619\ m/s}$

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d) $\vec{J} = \Delta\vec{p} = m_2(\vec{v}_f - \vec{v}_2) \longrightarrow \vec{J} = m_2(-13.619\,m/s - -36.75\,m/s)\hat{i}$

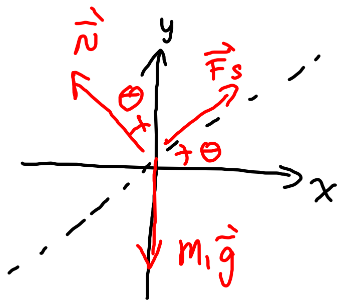
By conservation of momentum

$m_1 v_1 \hat{i} - m_2 v_2 \hat{i} = (m_1 + m_2)\vec{v}_f$

$\Rightarrow \vec{v}_f = \frac{m_1 v_1 - m_2 v_2}{(m_1 + m_2)}\hat{i} = -13.619\,m/s\ \hat{i}$

$\Rightarrow \boxed{\vec{J} = +4.63\ kg\,m/s\ \hat{i}}$

2). a).



Here,

$a_1 = a_2$ since the block is stationary relative to the inclined plane.

b). Along x

$$|\vec{N}|\sin\theta - |\vec{F}_s|\cos\theta = m_1 a_1$$

Along y

$$|\vec{N}|\cos\theta + |\vec{F}_s|\sin\theta - m_1 g = 0$$

Also,

$$|\vec{F}_s| = \mu_s |\vec{N}|$$

Since the block is on the verge of slipping.

$$\Rightarrow \quad |\vec{N}|\cos\theta + \mu_s |\vec{N}|\sin\theta = m_1 g \Rightarrow |\vec{N}|\left[\cos\theta + \mu_s \sin\theta\right] = m_1 g$$

$$\Rightarrow \quad |\vec{N}| = \frac{m_1 g}{\left[\cos\theta + \mu_s \sin\theta\right]} = \frac{0.4\,\text{kg}\,(9.8\,\text{m/s}^2)}{\left[\cos(50^\circ) + 0.3\sin(50^\circ)\right]} = 4.49\,\text{N}$$

2) c). $|\vec{N}|\sin\theta - \mu_s|\vec{N}|\cos\theta = m_1 a_1$

$$\Rightarrow \quad a_1 = \frac{|\vec{N}|\left[\sin\theta - \mu_s\cos\theta\right]}{m_1}$$

$$\Rightarrow \quad a_1 = \frac{(4.49\,N)\left[\sin(50^\circ) - 0.3\cos(50^\circ)\right]}{0.4\,kg}$$

$$\Rightarrow \quad \boxed{a_1 = 6.44\ m/s^2}$$

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d). $\text{Work} = \int \vec{F}_{net} \cdot dx\,\hat{i}$ Here, $\vec{F}_{net}$ is constant.

so, $\text{Work} = |\vec{F}_{net}|\Delta x = m_1 a_1 d$

$$\Rightarrow \quad \boxed{\text{Work} = 0.4\,kg\,(6.44\ m/s^2)(5\,m) = 12.875\ J}$$

3). a). $T = 2\pi\sqrt{\frac{L}{a}}$, Here a is the acceleration due to gravity near the surface of the planet.

$$\Rightarrow \left(\frac{T}{2\pi}\right)^2 = \frac{L}{a} \Rightarrow a = \frac{L}{\left(\frac{T}{2\pi}\right)^2} = \left(\frac{2\pi}{T}\right)^2 L$$

$$\Rightarrow a = \left(\frac{2\pi}{3.3s}\right)^2 (1m) \Rightarrow \boxed{a = 3.625 \, m/s^2}$$

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b). Here,

$$v_{f_y}^2 = v_{i_y}^2 - 2a\Delta y$$

$$\Rightarrow 0 = (v_i \sin\theta)^2 - 2ah \Rightarrow v_i^2 \sin^2\theta = 2ah$$

$$\Rightarrow \boxed{v_i = \frac{\sqrt{2ah}}{\sin\theta} = \frac{\sqrt{2(3.625 \, m/s^2)(8m)}}{\sin(30°)} = 15.23 \, m/s}$$

3). c). For the acceleration due to gravity at the surface of a spherical planet of mass M & radius R

$$a = \frac{GM}{R^2}$$

So,

$$a = \frac{GM_P}{(0.5R_E)^2} \quad \& \quad g = \frac{GM_E}{R_E^2}$$

$$\Rightarrow \quad M_P = \frac{\frac{a}{4}R_E^2}{G} \quad \& \quad M_E = \frac{gR_E^2}{G}$$

$$\Rightarrow \quad \boxed{\frac{M_P}{M_E} = \frac{a\,\cancel{R_E^2}}{4\cancel{G}}\,\frac{\cancel{G}}{g\,\cancel{R_E^2}} = \frac{a}{4g} = \frac{3.625\,m/s^2}{4(9.8\,m/s^2)} = 0.092}$$

3). d). In order for the golf ball to escape the planet, its total energy at the surface of the planet must be at least equal to zero.

So,

$$\frac{1}{2} m v_{min}^2 - \frac{G m M_p}{(0.5 R_E)} = 0$$

$$\Rightarrow \frac{1}{2} \cancel{m} v_{min}^2 = \frac{G \cancel{m} M_p}{(0.5 R_E)} \Rightarrow v_{min} = \sqrt{\frac{4 G M_p}{R_E}} = \sqrt{\frac{4G(0.092)M_E}{R_E}}$$

Note, if the golf ball were on Earth.

$$\frac{1}{2} m v^2 - \frac{G m M_E}{R_E} = 0 \Rightarrow \frac{1}{2} \cancel{m} v^2 = \frac{G \cancel{m} M_E}{R_E}$$

$$= \sqrt{2(0.092)} \sqrt{\frac{2G M_E}{R_E}}$$

$$\Rightarrow v = \sqrt{\frac{2G M_E}{R_E}}$$

$$v_{min} = \sqrt{2(0.092)}\, v = \sqrt{2(0.092)}\,(11.2 \text{ km/s})$$

$$\Rightarrow \boxed{v_{min} = 4.8 \text{ km/s}}$$

← Numerical answer is fairly sensitive to rounding.

4). a). Using the table given in the exam, the parallel-axis theorem and the definition of moment of inertia of a point particle.

$$I = \frac{M_R D^2}{12} + M_R\left(\frac{D}{2}\right)^2 + m_p\left(\frac{D}{2}\right)^2$$

$$\Rightarrow I = M_R D^2\left[\frac{1}{12} + \frac{1}{4}\right] + \frac{m_p D^2}{4}$$

$$\Rightarrow I = M_R D^2 \frac{4}{12} + \frac{m_p D^2}{4} = D^2\left[\frac{M_R}{3} + \frac{M_p}{4}\right]$$

$$\Rightarrow I = (2\,m)^2\left[\frac{3\,kg}{3} + \frac{1.5\,kg}{4}\right]$$

$$\Rightarrow \boxed{I = 5.5\ kg\,m^2}$$

4). b). Here, the angular momentum relative to the pivot point is conserved.

$$\vec{L}_i = \frac{D}{2} m_p v_0 \hat{k} \quad \& \quad \vec{L}_f = I \omega \hat{k}$$

$$\Rightarrow \frac{D}{2} m_p v_0 = I\omega \Rightarrow \omega = \frac{D m_p v_0}{2I}$$

$$\Rightarrow \boxed{\omega = \frac{(2\,m)(1.5\,kg)(1.2\,m/s)}{2(5.5\,kg\,m^2)} = 0.3273\,rad/s}$$

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c). $$KE_f = \frac{1}{2} I \omega^2 = \frac{1}{2}(5.5\,kg\,m^2)(0.3273\,rad/s)^2$$

$$\Rightarrow \boxed{KE_f = 0.2945\,J}$$

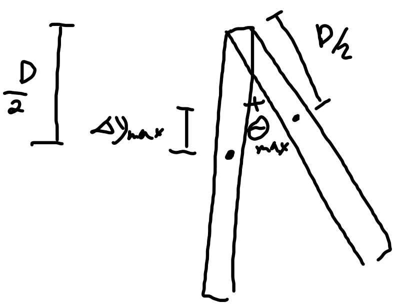
4). d). After the collision, the mechanical energy of the "rod-particle-Earth" system is conserved.

So,

$$\frac{1}{2} I \omega^2 = (m_R + m_P)\, g\, \Delta y_{max}$$

Here Δy_{max} is the maximum change in y of the center of mass of the "rod-particle" system.

$$\Rightarrow \Delta y_{max} = \frac{I\omega^2}{2(m_R+m_P)g} = \frac{0.2945\,\text{J}}{(3\text{kg}+1.5\text{kg})(9.8\,\text{m/s}^2)} = 0.006679\,\text{m}$$



$$\Rightarrow \frac{D}{2}\cos\theta_{max} + \Delta y_{max} = \frac{D}{2}$$

$$\Rightarrow \frac{D}{2}\left[1-\cos\theta_{max}\right] = \Delta y_{max} \Rightarrow 1-\cos\theta_{max} = \frac{2\Delta y_{max}}{D}$$

$$\Rightarrow \cos\theta_{max} = 1 - \frac{2\Delta y_{max}}{D} = 0.99332$$

$$\Rightarrow \boxed{\theta_{max} = 6.626^\circ}$$